KERALA CEE

Engineering Entrance Exam

Solved Paper 2021

Physics

1. When two sound waves of slightly different frequencies f_1 and f_2 are sounded together, then the time interval between successive maxima is

(a) $\frac{1}{f_1 + f_2}$ (b) $\frac{1}{f_1} + \frac{1}{f_2}$

(c) $\frac{1}{f_1 - f_2}$ (d) $\frac{1}{f_1 f_2}$

(e) $\frac{1}{f_1} - \frac{1}{f_2}$

2. The electric potential at a point at a distance r due to an electric dipole is proportional to

(a) r^2 (b) r (c) r^{-1} (d) r^{-2}

(e) r^{-3}

3. An air capacitor and identical capacitor filled with dielectric medium of dielectric constant 5 are connected in series to a voltage source of 12V. The fall of potential across C_1 and C_2 are respectively

(a) 2 V and 10 V (b) 10 V and 2 V

(c) 6 V and 6 V (d) 4 V and 8 V

(e) 8 V and 4 V

4. The ratio of the magnitudes of electrostatic force between two protons at a distance r apart to that between two electrons at the same distance of separation is

(a) 1 : 1 (b) 2 : 1

(c) 1 : 2 (d) 4 : 1

(e) 1 : 4

5. When two charges are kept in air medium, at certain distance d apart, the force between them is F. When they are kept in a dielectric medium at the same distance of separation, the force between them becomes $F/2$. Then, the dielectric constant of the medium is

(a) 5 (b) 2 (c) 4 (d) 3

(e) 8

6. The magnitude of the drift velocity per unit electric field is defined as

(a) mobility (b) resistivity

(c) conductivity (d) current density

(e) impedance

7. A Wheatstone network is balanced with respective resistors $5\,\Omega, 10\,\Omega, 20\,\Omega$ and $40\,\Omega$ in the P, Q, R and S arms. If a $40\,\Omega$ resistor is connected across S arm, then the bridge is again balanced by connecting

(a) 10Ω across R (b) 10Ω across P

(c) 20Ω across Q (d) 20Ω across P

(e) 10Ω across Q

8. If one cell is connected wrongly in a series combination of four cells each of emf 1.5 V and internal resistance of 0.5 Ω, then the equivalent internal resistance of the combination is

(a) 0.5Ω (b) 1Ω

(c) 1.5Ω (d) $2\,\Omega$

(e) 2.5Ω

9. A carbon resistor is marked with the rings coloured blue, black, red and silver. Its resistance (in ohm) is
(a) $60 \times 10^2 \pm 10\%$ (b) $1 \times 10^5 \pm 10\%$
(c) $1 \times 10^6 \pm 5\%$ (d) $3.2 \times 10^4 \pm 5\%$
(e) $4.5 \times 10^2 \pm 5\%$

10. A conductor of length 20 cm carrying a current of 5 A is placed at an angle of 30° to the external magnetic field of 0.5 T. The force acting on it is
(a) 0.5 N (b) 5 N
(c) 0.25 N (d) 2.5 N
(e) 0.125 N

11. A current carrying coil placed in a magnetic field B experience a torque τ. If θ is the angle between the normal to the plane of the coil and field B and ϕ is the flux linked with the coil, then
(a) τ is minimum for $\theta = 90°$
(b) τ and ϕ are maximum for $\theta = 0°$
(c) ϕ is maximum for $\theta = 90°$
(d) τ and ϕ are zero for $\theta = 90°$
(e) τ is zero and ϕ is maximum for $\theta = 0°$

12. In cyclotron, the frequency of revolution of the charged particle in a magnetic field is independent of
(a) its mass
(b) its energy
(c) oscillatory frequency
(d) magnetic field
(e) its charge

13. The hard ferromagnetic material among the following is
(a) gadolinium (b) iron
(c) cobalt (d) alnico
(e) nickel

14. If B_c is the magnetic induction at the centre of a circular coil carrying current, then the magnetic induction at a point on the axis of the coil at a distance equal to the radius of the coil is
(a) $\frac{B_c}{2\sqrt{2}}$ (b) $\frac{B_c}{2}$
(c) $\frac{B_c}{4}$ (d) $\frac{B_c}{\sqrt{2}}$
(e) $\frac{B_c}{8}$

15. If air core is replaced by an iron core is an inductor, its self-inductance is increased from 0.02 mH to 40 mH. The relative permeability of iron is
(a) 5000 (b) 2000 (c) 200 (d) 500
(e) 400

16. Among various circuits constructed with resistor R, inductor L and capacitor C, the circuit that gives maximum power dissipation is
(a) purely inductive circuit
(b) purely capacitive circuit
(c) purely resistive circuit
(d) L-C series circuit
(e) C-R series circuit

17. Eddy currents are not used in the application of
(a) induction furnace
(b) thermal generators
(c) electromagnetic damping
(d) electric power meters
(e) magnetic braking in trains

18. The total intensity of earth's magnetic field at the poles is 7 units. Its value at the equator is
(a) $7\sqrt{2}$ units (b) 3.5 units
(c) 7 units (d) $\frac{7}{\sqrt{2}}$ units
(e) 14 units

19. Electromagnetic wave against their detection devices are matched below. The mismatch is

(a)	Gamma rays	Ionisation chamber
(b)	Microwaves	Point contact diode
(c)	X-rays	Photographic film
(d)	Ultraviolet rays	Thermopiles
(e)	Infrared rays	Bolometer

20. In an electromagnetic wave, the oscillating electric and magnetic field vectors are oriented in
(a) mutually perpendicular directions with a phase difference of $\pi/2$
(b) the same direction and in the same phase
(c) mutually perpendicular directions with a phase difference of π
(d) the same direction with a phase difference of $\pi/2$
(e) mutually perpendicular directions and are in phase

21. Fresnel distance for an aperture of size a illuminated by a parallel beam of light of wavelength λ, deciding the validity of ray optics is

(a) $\frac{\lambda}{a^2}$ (b) λa (c) $a^2\lambda$ (d) $\frac{a^2}{\lambda}$
(e) $a^2\lambda^2$

22. The apparent depth of a needle lying in a water beaker is found to be 9 cm. If water is replaced by a liquid of refractive index 1.5, then the apparent depth of needle will be (μ of water is 4/3)

(a) 10 cm (b) 9 cm (c) 12 cm (d) 7 cm
(e) 8 cm

23. An object is placed at 10 cm in front of a concave mirror. If the image is at 20 cm from the mirror on the same side of the object, then the magnification produced by the mirror is

(a) 3 (b) −0.5
(c) −2 (d) 0.33
(e) −1

24. In Young's double-slit experiment, two different light beams of wavelengths λ_1 and λ_2 produce interference pattern with band widths β_1 and β_2, respectively. If the ratio between β_1 and β_2 is 3 : 2, then the ratio between λ_1 and λ_2 is

(a) 3 : 1 (b) 1 : 3
(c) 2 : 3 (d) 3 : 2
(e) 4 : 5

25. If θ_p is the polarising angle for a glass plate of refractive index μ and critical angle θ_c, then

(a) $\theta_p = \theta_c$ (b) $\tan\theta_p . \sin\theta_c = 1$
(c) $\theta_p\theta_c = 1$ (d) $\tan\theta_p = \sin\theta_c$
(e) $\tan\theta_p \sin\theta_c = \mu$

26. Two materials A and B having respective work functions 3 eV and 4 eV are emitting photoelectrons of same maximum kinetic energy of 1 eV. If the wavelength of incident light of A is 500 nm, then that of light incident on B is

(a) 400 nm (b) 300 nm
(c) 350 nm (d) 600 nm
(e) 250 nm

27. If the momentum of an α-particle is half that of a proton, then the ratio between the wavelengths of their de-Broglie waves is

(a) 1 : 2 (b) 4 : 1
(c) 1 : 4 (d) 1 : 1
(e) 2 : 1

28. During β^- decay of a radioactive element, there is an increase in its

(a) mass number (b) neutron number
(c) electron number (d) proton number
(e) atomic weight

29. 10^{18} fissions per second is required for producing power of 300 MW in a nuclear power station. To increase the power output to 360 MW, the additional number of fissions required per second is

(a) 2×10^{18} (b) 5×10^{18}
(c) 5×10^{17} (d) 6×10^{17}
(e) 2×10^{17}

30. The ratio of the total energy E of the electron to its kinetic energy K in hydrogen atom is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) −1
(e) $-\frac{1}{2}$

31. If the mass numbers of two nuclei are in the ratio 3 : 2 , then the ratio of their nuclear densities is

(a) $3^{1/2} : 2^{1/3}$ (b) $2^{1/3} : 3^{1/3}$
(c) 2 : 3 (d) 1 : 1
(e) 3 : 2

32. In p-type semiconductors,

(a) holes are minority carrier
(b) the vacancy of electron is a hole with negative charge
(c) the impurity element added in donor type
(d) for every pentavalent impurity atom added an extra hole is created
(e) the electron will move from one hole to another hole constituting a flow of current

33. In a CB mode of a transistor, the current through the emitter is 6 mA. If the current gain of the transistors is 0.95, then its base current is

(a) 0.2 mA (b) 0.3 mA (c) 0.5 mA (d) 0.4 mA
(e) 0.8 mA

34. The compound semiconductor used for making LEDs of different colours is

(a) gallium arsenide phosphide
(b) indium arsenide phosphide
(c) indium arsenide selenide
(d) gallium arsenide selenide
(e) scandium arsenide phosphide

35. A transistor amplifier along with a tank circuit with positive feedback will act as

(a) power amplifier (b) voltage amplifier
(c) full wave rectifier (d) half-wave rectifier
(e) oscillator

36. In a transmitter, the audio signal of frequency ω_m is modulated by the carrier signal ω_c and the band pass filter in it rejects the frequencies

(a) ω_c and ω_m (b) $\omega_c - \omega_m$ and $\omega_c + \omega_m$
(c) ω_m and $2\omega_c$ (d) $\omega_c - \omega_m$ and ω_c
(e) $\omega_c + \omega_m$ and ω_c

37. Pick out the incorrect statements from the following.

(a) Speech signal required a bandwidth of 2800 Hz
(b) The approximate bandwidth to transmit music is 20 kHz
(c) The bandwidth of video signals required to transmit pictures is 4.2 MHz
(d) The bandwidth usually allocated to transmit TV signals is 6 MHz
(e) Digital signals are usually in the form of sine waves

38. A physical quantity A on multiplication with velocity results in another quantity B. If the quantity B is energy, then the quantity A is

(a) mass (b) momentum
(c) force (d) acceleration
(e) power

39. If the percentage errors in the measurements of mass, length and time are 1%, 2% and 3% respectively, then the maximum permissible error in the measurement of the acceleration of a particle is

(a) 8% (b) 9% (c) 6% (d) 10%
(e) 2%

40. The radius of a circular plate is 1.05 m. Its area (in m^2) upto correct significant figures is

(a) 3.47 (b) 3.475 (c) 3.467 (d) 3.82
(e) 3.825

41. The velocity of a moving particle at any instant is $\hat{\mathbf{i}} + \hat{\mathbf{j}}$. The magnitude and direction of the velocity of the particle are

(a) 2 units and 45° with the X-axis
(b) 2 units and 30° with the Z-axis
(c) $\sqrt{2}$ units and 45° with the X-axis
(d) $\sqrt{2}$ units and 60° with the Y-axis
(e) 2 units and 60° with the X-axis

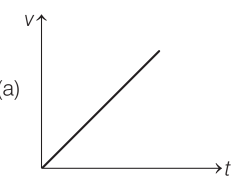
42. A hammer is dropped into a mine. Its velocities at depths d, $2d$ and $3d$ are in the ratio

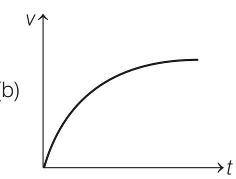
(a) 1 : 2 : 3
(b) $1 : \sqrt{2} : \sqrt{3}$
(c) 1 : 4 : 9
(d) 6 : 3 : 2
(e) 1 : 1 : 1

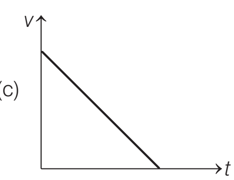
43. The stopping distance of a moving vehicle is proportional to the

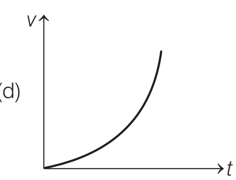
(a) initial velocity
(b) cube of the initial velocity
(c) square of the initial velocity
(d) cube root of the initial velocity
(e) square root of the initial velocity

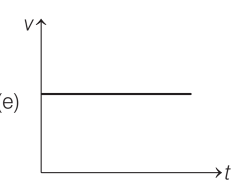
44. When a body starts from rest and moves with a constant acceleration, the velocity-time graph for its motion is

(a)


(b)


(c)


(d)


(e)


45. A wooden block of mass 10 kg is moving with an acceleration of 3 ms^{-2} on a rough floor. If the coefficient of friction is 0.3, then the applied force on it is ($g = 10\ ms^{-2}$)

(a) 10 N (b) 30 N (c) 80 N (d) 60 N
(e) 65 N

46. Which one of the following statement is incorrect?

(a) The state of rest or uniform linear motion both imply zero acceleration.
(b) A net force is needed to keep a body in uniform motion.
(c) Inertia means resistance to change.
(d) The rate of change of momentum is proportional to the applied force.
(e) Momentum is a vector quantity.

47. On a conveyor belt moving with a speed u, sand falls at a constant rate $\left(\frac{dm}{dt}\right)$, where m is the mass of sand. The extra force required to maintain the speed of the belt is

(a) $m\left(\frac{du}{dt}\right)$ (b) mu
(c) $\left(\frac{dm}{dt}\right)/u$ (d) $u\left(\frac{dm}{dt}\right)$
(e) $\frac{1}{m}\left(\frac{du}{dt}\right)$

48. Area under the force-time graph gives the change in

(a) velocity
(b) acceleration
(c) linear momentum
(d) angular momentum
(e) impulsive force

49. When a metal spring is elongated within its elastic limit,

(a) work is done by the spring
(b) potential energy is stored in it
(c) its potential energy is lost
(d) its total energy remains constant
(e) its kinetic energy is increased

50. The instantaneous power in terms of force F and instantaneous velocity v is

(a) $P = F \cdot t$ (b) $P = F \cdot v$
(c) $P = F \cdot v^{-1}$ (d) $P = F \cdot v^{-2}$
(e) $P = F \cdot v \cdot t^{-1}$

51. A ball with 10^3 J of kinetic energy collides with a horizontally mounted springs. If the maximum compression of the spring is 50 cm, then the spring constant of the spring is

(a) 2×10^3 Nm^{-1} (b) 6×10^3 Nm^{-1}
(c) 8×10^3 Nm^{-1} (d) 5×10^3 Nm^{-1}
(e) 3×10^3 Nm^{-1}

52. An object released from certain height h from the ground, rebounds to a height $\frac{h}{4}$ after striking the ground. The fraction of the energy lost by it is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$
(e) $\frac{3}{8}$

53. A solid metal ring and a disc of same radius and mass are rotating about their diameters with same angular frequency. The ratio of their respective rotational kinetic energy values is

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4
(e) 4 : 1

54. The x and y-coordinates of the particles of masses m, $2m$ and $3m$ are respectively $(0, 0)$, $(1, 0)$ and $(-2, 0)$. The x-coordinate of the centre of mass of the system is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$
(e) $\frac{1}{6}$

55. Radius of gyration of a solid cylinder of radius R and length L about its long axis of symmetry is

(a) R (b) $\frac{R}{\sqrt{2}}$ (c) $\sqrt{2}R$ (d) $\frac{R}{2}$
(e) $2R$

56. When no external torque acts on a rotating system, then

(a) angular momentum of the system is not conserved
(b) its rotational kinetic energy is conserved
(c) its rotational kinetic energy is independent of moment of inertia
(d) its rotational kinetic energy is directly proportional to moment of inertia
(e) its rotational kinetic energy is inversely proportional to moment of inertia

57. If T be the time period of a planet around the sun and d is its mean distance from the sun, then according to Kepler's third law,

(a) $T \propto d$ (b) $T \propto d^2$ (c) $T^2 \propto d^3$ (d) $T^2 \propto d$
(e) $T^2 \propto d^{-3}$

58. If the earth shrinks to half of its present size and its mass reduces to half of its actual mass, then the acceleration due to gravity (g) on its surface will be

(a) $4g$ (b) g (c) $2g$ (d) $\frac{g}{2}$
(e) $3g$

59. When two identical spheres each of radius r are kept in contact with each other, then the force of attraction between the two spheres is proportional to

(a) r^2 (b) r^4 (c) r^6 (d) r^{-2}
(e) r^{-4}

60. With the increase of temperature,

(a) surface tension of liquid increases
(b) viscosity of gases decrease
(c) viscosity of liquids increase
(d) Both the surface tension and viscosity of liquids increase
(e) Both the surface tension and viscosity of liquids decrease

61. The true statement is

(a) Young's modulus of a wire depends on its length
(b) The unit of Young's modulus is Nm^{-1}
(c) Dimensional formula of stress is same as that of force
(d) The unit of strain is kgm^{-2}
(e) Compressibility is the reciprocal of bulk modulus

62. When a body is strained, energy stored per unit volume is (Y = Young's modulus)

(a) $\frac{\text{(stress)}}{Y}$ (b) $\frac{Y \times \text{strain}}{2}$
(c) $\frac{\text{(stress)}^2}{2Y}$ (d) $Y \times \text{(strain)}^2$
(e) $\frac{1}{2}\left(\frac{\text{stress}}{Y}\right)$

63. According to equation of continuity, when a liquid flows through a tube of variable cross-section a with variable velocity v, the quantity remains constant is

(a) av^2 (b) a^2v (c) av (d) $\frac{a}{v}$
(e) $\frac{a^2}{v}$

64. Two thermally insulated identical vessels A and B are connected through a stopcock. A contains a gas at STP and B is completely evacuated. If the stopcock is suddenly opened, then

(a) temperature is halved
(b) internal energy of the gas is halved
(c) internal energy of the gas and pressure are halved
(d) temperature and internal energy of the gas remain the same
(e) pressure and internal energy of the gas remain the same

65. A process in which there is no flow of heat between the system and surroundings is a/an

(a) adiabatic process (b) cyclic process
(c) isobaric process (d) isochoric process
(e) isothermal process

66. When the temperature of the source of Carnot engine is at 400 K, its efficiency is 25%. The required increase in temperature of the source to increase the efficiency to 50% is

(a) 800 K (b) 600 K
(c) 100 K (d) 400 K
(e) 200 K

67. When a ideal diatomic gas is heated at constant pressure, fraction of heat energy supplied that increases the internal energy of the gas is

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$
(c) $\frac{3}{5}$ (d) $\frac{5}{3}$
(e) $\frac{2}{3}$

68. The ratio of the kinetic energy values of 4g of hydrogen (H_2) to 7g of nitrogen (N_2) at room temperature is

(a) 4 : 1 (b) 1 : 4
(c) 4 : 7 (d) 7 : 4
(e) 1 : 1

69. A planet with radius R and acceleration due to gravity g, will have atmosphere only, if rms speed of air molecules is less than

(a) $1.414\sqrt{gR}$ (b) $1.732\sqrt{gR}$
(c) $2\sqrt{gR}$ (d) $3.14\sqrt{gR}$
(e) $2.75\sqrt{gR}$

70. If the ratio of the acceleration due to gravity on the surface of earth to that on the surface of the moon is 6 : 1, then the ratio of the period of a simple pendulum on their surfaces is

(a) 1 : 1 (b) 1 : 6
(c) 1 : 3 (d) $1:\sqrt{6}$
(e) $1:\sqrt{3}$

71. The velocity of a transverse wave propagating on a stretched string represented by the equation

$y = 0.5\sin\left(\frac{\pi}{2}t + \frac{\pi}{3}x\right)$ is

(where, x and y are in metres and t in seconds)

(a) 0.5 ms^{-1} (b) 1.0 ms^{-1}
(c) 2 ms^{-1} (d) 3 ms^{-1}
(e) 1.5 ms^{-1}

72. The kinetic energy of a particle of mass (m) executing linear simple harmonic motion with angular velocity ω and amplitude a is $\frac{1}{4}ma^2\omega^2$ at a distance of ……… from the mean position.

(a) $\frac{a}{\sqrt{2}}$ (b) $\frac{a}{2}$ (c) $\frac{a}{4}$ (d) a
(e) $\frac{a}{8}$

73. The reagent that is used to convert but-2-yne to *trans*-but-2-ene is

(a) H_2/Pd/C (b) $NaBH_4$
(c) Sn/HCl (d) Na/liquid NH_3
(e) Zn-Hg/HCl

74. Compound 'A' is obtained by the reaction of benzyl chloride with magnesium metal in dry ether followed by treatment with water. What is the compound 'A'?

(a) Toluene (b) Benzyl alcohol
(c) Phenol (d) Benzene
(e) Benzaldehyde

75. The correct increasing order of boiling points of the following compounds is

(a) $CH_2Br_2 < CH_3Br < CHBr_3 < CH_3Cl$
(b) $CH_2Br_2 < CHBr_3 < CH_3Br < CH_3Cl$
(c) $CH_3Cl < CH_3Br < CH_2Br_2 < CHBr_3$
(d) $CH_3Cl < CHBr_3 < CH_3Br < CH_2Br$
(e) $CHBr_3 < CH_2Br_2 < CH_3Br < CH_3Cl$

76. Compound 'A', 'B' and 'C' have the same molecular formula C_7H_8O. Compound 'A' and 'B' liberate hydrogen gas with sodium metal. When treated with sodium hydroxide, compound 'B' alone dissolves. Compound 'C' is inert towards both sodium metal and sodium hydroxide. Compound 'A', 'B' and 'C' are respectively

(a) cresol, benzyl alcohol and anisole
(b) benzyl alcohol, cresol and anisole
(c) benzyl alcohol, anisole and cresol
(d) cresol, anisole and benzyl alcohol
(e) anisole, cresol and benzyl alcohol

77. The suitable Grignard reagent used for the preparation of 2-methylpropan-1-ol using methanal is

(a) $CH_3—CH_2—CH_2MgBr$
(b) $CH_3—CH_2—CH_2—CH_2MgBr$
(c) $CH_3—CH(CH_3)—CH_2MgBr$
(d) $(CH_3)_3C—MgBr$
(e) $CH_3—CH(CH_3)—MgBr$

78. Isopropylbenzene (cumene) is oxidised in the presence of air to give compound 'X' which on hydrolysis in the presence of acids gives compounds 'Y' and 'Z'. Compounds 'X', 'Y' and 'Z' are respectively

(a) benzyl alcohol, benzaldehyde, ethanol
(b) cumene hydroperoxide, phenol, acetaldehyde
(c) cumene hydroperoxide, benzaldehyde, acetone
(d) cumene hydroperoxide, phenol, acetone
(e) cumene hydroperoxide, benzaldehyde, acetaldehyde

79. A research scholar returned to the laboratory after the lock down due to Covid-19. He kept acetone, benzaldehyde, acetaldehyde and diethyl ketone in four different bottles.The bottles contained only the label as P, Q, R and S. He forgot which bottle contained which compound. Compounds P and R only underwent iodoform test. Compound R alone gave reddish brown precipitate with

Fehling's reagent. Compounds Q and R alone underwent Tollen's test. Compound S did not answer any of the above tests.

Identify the compounds P, Q, R and S.

(a) P-diethyl ketone; Q-benzaldehyde; R-acetaldehyde; S-acetone
(b) P-acetone; Q-benzaldehyde; R-acetaldehyde; S-diethyl ketone
(c) P-acetone; Q-acetaldehyde; R-benzaldehyde S-diethyl ketone
(d) P-acetaldehyde; Q-acetone; R-diethyl ketone; S-benzaldehyde
(e) P-benzaldehyde; Q-diethyl ketone; R-aetone; S-acetaldehyde

80. The increasing order of acid strength of the following carboxylic acids is

(a) $ClCH_2—CH_2—COOH < ClCH_2COOH < NC—CH_2COOH < CHCl_2COOH$
(b) $ClCH_2—COOH < NC—CH_2COOH < ClCH_2CH_2COOH < CHCl_2COOH$
(c) $ClCH_2—CH_2—COOH < CHCl_2—COOH < ClCH_2—COOH < NC—CH_2—COOH$
(d) $NC—CH_2—COOH < Cl—CH_2COOH < CH—Cl_2COOH < Cl—CH_2CH_2COOH$
(e) $ClCH_2CH_2—COOH < CHCl_2COOH < ClCH_2COOH < NC—CH_2COOH$

81. Which one of the following is not correct with respect to properties of amines?

(a) pK_b of aniline is more than that of methylamine.
(b) Ethylamine is soluble in water whereas aniline is not.
(c) Ethanamide on reaction with Br_2 and NaOH gives ethylamine.
(d) Ethylamine reacts with nitrous acid to give ethanol.
(e) Aniline does not undergo Friedel-Craft's reaction.

82. The increasing order of extent of H-bonding of the alkyl ammonium ions, RNH_3^+, $R_2NH_2^+, R_3NH^+$ in water is

(a) $R_3NH^+ < R_2NH_2^+ < RNH_3^+$
(b) $R_3NH^+ < RNH_3^+ < R_2NH_2^+$
(c) $R_2NH_2^+ < RNH_3^+ < R_3NH^+$
(d) $RNH_3^+ < R_2NH_2^+ < R_3NH^+$
(e) $RNH_3^+ < R_3NH^+ < R_2NH_2^+$

83. The conversion of benzene diazonium chloride to bromobenzene by treating with HBr in the presence of copper powder is called

(a) Sandmeyer reaction
(b) Gattermann reaction
(c) Wurtz reaction
(d) Hofmann reaction
(e) Gabriel synthesis

84. Which one of the following statements is true with regard to glucose?

(a) It gives Schiff's test
(b) It forms addition product with $NaHSO_3$
(c) Its pentaacetate does not react with NH_2OH
(d) It does not undergo mutarotation
(e) β-form of glucose is obtained by crystallisation from conc. solution of glucose at 303 K

85. Fibrous protein present in muscles is

(a) keratin
(b) albumin
(c) insulin
(d) myosin
(e) histidine

86. The drug used to inhibit the enzymes which catalyse the degradation of noradrenaline is

(a) phenelzine
(b) prontosil
(c) cimetidine
(d) terfenadine
(e) chloramphenicol

87. The gas which is the major contributor to global warming is

(a) NO_2
(b) CO_2
(c) SO_2
(d) O_2
(e) N_2O

88. A cooking gas contains carbon and hydrogen only. A volume of 11.2 L of this gas is found to weigh 22 g at STP. Then the molecular formula of the gas is

(a) C_3H_8
(b) C_2H_2
(c) C_2H_4
(d) C_2H_6
(e) C_3H_4

89. The number of electrons in an atom that may have the quantum numbers $n=3$ and $m_s=+\frac{1}{2}$ is

(a) 32 (b) 9 (c) 18 (d) 16
(e) 8

90. "No two electrons is an atom can have the same set of four quantum numbers." This is known as
(a) Hund's rule
(b) Pauli's exclusion principle
(c) Aufbau principle
(d) Heisenberg's principle
(e) Fajan's rule

91. The first ionisation enthalpy is the least in
(a) germanium (b) antimony
(c) tellurium (d) arsenic
(e) bismuth

92. Predict in which of the following, entropy decreases
(a) A liquid crystallises into a solid
(b) Temperature of a crystalline solid is raised from 0K to 115 K.
(c) $2NaHCO_3(s) \longrightarrow Na_2CO_3(s) + CO_2(g) + H_2O(g)$
(d) $H_2(g) \longrightarrow 2H(g)$
(e) $2SO_3(g) \longrightarrow 2SO_2(g) + O_2(g)$

93. In which one of the following, sp^2 hybridisation is involved in the central atom?
(a) NH_3 (b) BCl_3
(c) ClF_3 (d) PCl_3
(e) PH_3

94. In which one of the following molecules, the central atom has expanded octet?
(a) Sulphur dichloride (b) Boron trichloride
(c) Nitrogen dioxide (d) Ozone
(e) Sulphuric acid

95. A cyclic tube will burst if the volume of air inside exceeds 1L at the room temperature. If at 1 bar pressure the air occupies 500 mL, then up to what pressure can the tube be expanded at the same temperature?
(a) 2 bar (b) 1.5 bar
(c) 0.5 bar (d) 0.002 bar
(e) 1.2 bar

96. The ratio of the actual molar volume of gas to the ideal molar volume is .... of the gas.
(a) co-volume
(b) van der Waals' factor '*a*'
(c) critical volume
(d) molar gas constant
(e) compressibility factor

97. Enthalpy change is always negative for which one of the following processes?
(a) Enthalpy of ionisation
(b) Enthalpy fo sublimation
(c) Enthalpy of vapourisation
(d) Enthalpy of fusion
(e) Enthalpy of combustion

98. The enthalpy change of the evaporation of a liquid at its boiling point 127°C is $+40.32$ kJ mol^{-1}. What is the value of internal energy change for the above process at 127°C? ($R = 8.3$ JK^{-1} mol^{-1})
(a) -37.0 kJ mol^{-1} (b) $+43.0$ kJ mol^{-1}
(c) $+37.0$ kJ mol^{-1} (d) -43.0 kJ mol^{-1}
(e) $+43.64$ kJ mol^{-1}

99. In which one of the following equilibria Δn_g value is zero?
(a) $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$
(b) $Ni(s) + 4CO(g) \rightleftharpoons Ni(CO)_4(g)$
(c) $CO_2(g) + C(s) \rightleftharpoons 2CO(g)$
(d) $H_2(g) + Br_2(g) \rightleftharpoons 2HBr(g)$
(e) $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

100. The following concentrations were obtained for the formation of $NH_3(g)$ from $N_2(g)$ and $H_2(g)$ at equilibrium and at 500 K : $[N_2] = 1 \times 10^{-2}$ M, $[H_2] = 2 \times 10^{-2}$ M and $[NH_3] = 2 \times 10^{-2}$ M. The equilibrium constant, K_c, for the reaction
$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ at 500 K is
(a) 5×10^3 mol^{-2} dm^6 (b) 1×10^3 mol^{-2} dm^6
(c) 5×10^{-3} mol^{-2} dm^6 (d) 2×10^3 mol^{-2} dm^6
(e) 2×10^{-3} mol^{-2} dm^6

101. The SI unit of molar conductivity is
(a) S m^3 mol^{-1} (b) S m mol^{-1}
(c) S m mol^{-2} (d) S m^2 mol^{-1}
(e) S m^2 mol^{-2}

102. Which of the following is an example of disproportionation redox reaction?
(a) $N_2(g) + O_2(g) \longrightarrow 2NO(g)$
(b) $2H_2(g) + O_2(g) \longrightarrow 2H_2O(l)$
(c) $2Pb(NO_3)_2(s) \longrightarrow 2PbO(s) + 4NO_2(g) + O_2(g)$
(d) $NaH(s) + H_2O(l) \longrightarrow NaOH(aq) + H_2(g)$
(e) $2NO_2(g) + 2OH^- \longrightarrow NO_2^-(aq) + NO_3^-(aq) + H_2O(l)$

103. A scientist wants to perform an experiment in aqueous solution in a hill station where the boiling point of water is 98.98°C. How much urea (mol. wt. 60 g mol^{-1}) is to be added by him to 2 kg of water to get the boiling point 100°C at the same place?

(K_b of water = 0.51 $Kkgmol^{-1}$)

(a) 60 g (b) 120 g
(c) 180 g (d) 240 g
(e) 1.02 g

104. The vapour pressure of pure benzene at a certain temperature is 0.850 bar. A non-volatile, non-electrolyte solid weighing 1.0 g when added to 39.0 of benzene (molar mass 78 g mol^{-1}), vapour pressure of the solution is reduced to 0.845 bar. What is the molar mass of the solid substance?

(a) 340 g mol^{-1} (b) 170 g mol^{-1}
(c) 240 g mol^{-1} (d) 270 g mol^{-1}
(e) 370 g mol^{-1}

105. For the reaction, $2P + Q \rightleftharpoons P_2Q$, the rate of formation of P_2Q is 0.24 mol dm^{-3} s^{-1}.Then the rates of disappearance of P and Q respectively are

(a) – 0.48 mol dm^{-3} s^{-1} and – 0.48 mol dm^{-3} s^{-1}
(b) – 0.24 mol dm^{-3} s^{-1} and – 0.48 mol dm^{-3} s^{-1}
(c) – 0.48 mol dm^{-3} s^{-1} and – 0.24 mol dm^{-3} s^{-1}
(d) – 0.12 mol dm^{-3} s^{-1} and – 0.24 mol dm^{-3} s^{-1}
(e) – 0.24 mol dm^{-3} s^{-1} and – 0.12 mol dm^{-3} s^{-1}

106. Choose the correct set of reactions which follow first order kinetics

(i) Thermal decomposition of HI on gold surface.
(ii) Thermal decomposition of $N_2O_5(g)$ at constant volume.
(iii) Hydrogenation of ethene.
(iv) Decomposition of NH_3 on a hot Pt surface.
(v) Thermal decomposition of $SO_2Cl_2(g)$ at constant volume.

(a) (i), (ii), (iii) (b) (i), (iii), (iv)
(c) (i), (iv), (v) (d) (ii), (iv), (v)
(e) (ii), (iii), (v)

107. Which one of the following is true?

(a) Chemisorption is not specific in nature.
(b) Physisorption is irreversible.
(c) Both physisorption and chemisorption depend on the nature of the gas.
(d) Enthalpy of adsorption is high in physisorption.
(e) Chemisorption increases with surface area of adsorbent while in physisorption it is not.

108. When zinc metal is reacted with aqueous sodium hydroxide, the products formed are

(a) zinc hydroxide and oxygen only
(b) sodium zincate and oxygen only
(c) sodium zincate, hydrogen and oxygen
(d) sodium zincate and hydrogen only
(e) sodium zincate and hydrogen oxide only

109. '*syn*-gas' produced from sewage is a gaseous mixture of

(a) CH_4 and C_2H_6 (b) CO and H_2
(c) CO and CH_4 (d) CS_2 and CO
(e) CS_2 and CH_4

110. Choose the correct choice containing true statements regarding PCl_5.

(i) PCl_5 is prepared by the reaction of white phosphorus with excess of dry chrloine.
(ii) The complete hydrolysis of PCl_5 gives phosphoric acid.
(iii) PCl_5 has square pyramidal structure in gaseous phase.
(iv) All the five bonds in PCl_5 molecule are equivalent.

(a) (ii) and (iii) (b) (i) and (iii)
(c) (iii) and (iv) (d) (ii) and (iv)
(e) (i) and (ii)

111. Match the substances and their uses.

	Substances		Uses
A.	Silicones	(i)	Cracking of hydrocarbons
B.	Zeolites	(ii)	Light composite material for aircraft
C.	Quartz	(iii)	Flux for soldering metals
D.	Borax	(iv)	Water proofing of fabrics
E.	Boron fibres	(v)	Piezoelectric material

	A	B	C	D	E
(a)	(iv),	(ii),	(i),	(v),	(iii)
(b)	(i),	(ii),	(iv),	(iii),	(v)
(c)	(iv),	(i),	(iii),	(ii),	(v)
(d)	(iii),	(ii),	(i),	(iv),	(v)
(e)	(iv),	(i),	(v),	(iii),	(ii)

112. Choose the wrong statement in the following with regard to orthoboric acid

(a) It can be prepared by the hydrolysis of boron trihalide.
(b) It is not a protonic acid but acts as a Lewis acid.
(c) It has a layer structure.
(d) It is freely soluble in cold water.
(e) On heating above 370K it is forms first metaboric acid which on further heating yield B_2O_3.

113. The magnetic moment of a trivalent ion of a metal with Z =24 in aqueous solution is

(a) 3.87 BM (b) 2.84 BM
(c) 1.73 BM (d) 4.90 BM
(e) 5.92 BM

114. In the first row transition metals, the element that exhibits only +3 oxidation state is

(a) zinc (b) scandium
(c) nickel (d) titanium
(e) iron

115. The metal that has the highest melting point in the first series of transition element is

(a) titanium (b) vanadium
(c) chromium (d) iron
(e) manganese

116. In which one of the following complexes, the conductivity corresponds to 1 : 2 electrolyte in aqueous solution?

(a) Hexaamminecobalt(III) chloride
(b) Tetraamminedichlorocobalt(III) chloride
(c) Pentaamminechlorocobalt(III) chloride
(d) Triamminetriaquachromium(III) chloride
(e) Diamminesilver(I) dicyanoargentate(I)

117. The complex ion formed when the film developed in black and white photography is washed with hypo solution is

(a) $[Ag_2(S_2O_3)_2]^{3-}$ (b) $[Ag(S_2O_3)_2]^{3-}$
(c) $[Ag(S_2O_3)_2]^{3+}$ (d) $[Ag_2(S_2O_3)_2]^{3+}$
(e) $[Ag(S_2O_3)_3]^{3-}$

118. Which one of the following is an ore of aluminium?

(a) Kaolinite (b) Siderite
(c) Malachite (d) Calamine
(e) Haematite

119. In the estimation of nitrogen present in an organic compound, Kjeldahl's method cannot be applied to

(a) aniline (b) toluidine
(c) urea (d) pyridine
(e) benzylamine

120. Among the following the alkene that exhibits optical isomerism is

(a) 3-methyl-2-pentene
(b) 4-methyl-1-pentene
(c) 3-methyl-1-pentene
(d) 2-methyl-2-pentene
(e) 2, 3-dimethyl-2-butene

Mathematics

1. The set of all integer values of x that satisfy the inequality $19 \le -3x \le 27$ is

(a) $\{-9, -8, -7, -6\}$
(b) $\{-9, -6\}$
(c) $\{-9, -8, -7\}$
(d) $\{-9, -8, -7, \ldots, 4, 5, 6\}$
(e) $\varnothing$

2. Let X be the set $\{1, \pi, \{42, \sqrt{2}\}, \{1, 3\}\}$. Which of the following statement(s) is/are true?
$P: \pi \in X$, $Q: \{1, 3\} \subseteq X$ $R: \{1, \pi\} \subseteq X$

(a) P only (b) Q only
(c) R only (d) P and R
(e) P, Q and R

3. The value of θ in the range $0 \le \theta \le \frac{\pi}{2}$, which satisfies the equation $\sin\left(\theta + \frac{\pi}{6}\right) = \cos\theta$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{8}$
(e) $\frac{\pi}{5}$

4. If $\operatorname{cosec}\theta + \cot\theta = 5$, then the value of $\tan\theta$ is equal to

(a) $\frac{13}{24}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{1}{12}$
(e) $\frac{3}{12}$

5. The value of $\tan^{-1}\left(\frac{7}{4}\right) - \tan^{-1}\left(\frac{3}{11}\right)$ is equal to

(a) $\frac{-\pi}{3}$ (b) $\frac{-\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$
(e) π

6. If $0 < \theta < \frac{\pi}{2}$ and $\tan\theta = \frac{\sqrt{5}}{2}$, then $\cos\theta$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$
(e) $\frac{\sqrt{5}}{3}$

7. The value of $\sin^2\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ is equal to

(a) $\frac{4}{5}$ (b) $\frac{16}{25}$ (c) $\frac{9}{25}$ (d) $\frac{5}{3}$
(e) $\frac{25}{9}$

8. $\cos^4\frac{\pi}{12} - \sin^4\frac{\pi}{12}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) $\frac{\sqrt{3}-1}{2}$
(e) $\frac{\sqrt{2}}{2}$

9. $\tan\left(2\tan^{-1}\left(\frac{2}{5}\right)\right)$ is equal to

(a) $\frac{8}{5}$ (b) $\frac{10}{21}$ (c) $\frac{20}{21}$ (d) $\frac{21}{25}$
(e) $\frac{4}{25}$

10. The values of x in the interval $[0, \pi]$ such that $\sin 2x = \frac{\sqrt{3}}{2}$ are

(a) $\frac{\pi}{6}, \frac{\pi}{3}$ (b) $\frac{\pi}{6}, \frac{2\pi}{3}$
(c) $\frac{\pi}{3}, \frac{2\pi}{3}$ (d) $\frac{\pi}{6}, \frac{5\pi}{6}$
(e) $\frac{\pi}{3}, \frac{5\pi}{6}$

11. If $\sin\alpha + \sin\beta = \frac{\sqrt{6}}{2}$ and $\cos\alpha + \cos\beta = \frac{\sqrt{2}}{2}$, then $\cos(\alpha - \beta)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{-1}{2}$ (d) $\frac{-3}{2}$
(e) 0

12. If $ay = x + b$ is the equation of the line passing through the points $(-5, -2)$ and $(4, 7)$, then the value of $2a + b$ is equal to

(a) 1 (b) 3
(c) 5 (d) -3
(e) -1

13. The y-intercept of the line passing through $(2, 5)$ with slope $\frac{1}{2}$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

14. The equation of perpendicular bisector of the line segment joining the points $(10, 0)$ and $(0, -4)$ is

(a) $5x + 2y = 21$ (b) $5x + 2y = 0$
(c) $2x - 5y = 21$ (d) $5x - 2y = 21$
(e) $2x + 3y = 21$

15. The equation of the line which is parallel to $x + \frac{1}{2}y = \frac{3}{2}$ and passing through $(1, 3)$ is

(a) $2x + y = 7$ (b) $2x + y + 5 = 0$
(c) $2x + y = 3$ (d) $2x + y = 6$
(e) $2x + y = 5$

16. If x-intercept of the straight line $ax + 2ay = 30$ is 10, then the y-intercept is

(a) 5 (b) 10 (c) 15 (d) 20
(e) 30

17. A straight line makes an angle α with the positive direction of x-axis, where $\cos\alpha = \frac{\sqrt{3}}{2}$. If it passes through $(0, -2)$, then its equation is

(a) $\sqrt{3}x + y + 2 = 0$
(b) $\sqrt{3}y + x + 2 = 0$
(c) $\sqrt{3}y + x + 2\sqrt{3} = 0$
(d) $\sqrt{3}y - x + 2\sqrt{3} = 0$
(e) $\sqrt{3}x + y - 2\sqrt{3} = 0$

18. The equation of the circle is $3x^2 + 3y^2 + 6x - 4y - 1 = 0$. Then its radius is

(a) $\frac{1}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{16}{3}$
(e) $\frac{8}{3}$

19. The end-points of a diameter of a circle are $(-1, 4)$ and $(5, 4)$. Then the equation of the circle is

(a) $(x - 3)^2 + y^2 = 9$
(b) $(x - 3)^2 + (y + 4)^2 = 3$
(c) $(x - 2)^2 + (y - 4)^2 = 9$
(d) $(x + 3)^2 + (y + 4)^2 = 9$
(e) $(x - 3)^2 + (y - 4)^2 = 4$

20. The two diameters of a circle are segments of the straight lines $x - y = 5$ and $2x + y = 4$. If the radius of the circle is 5, then the equation of the circle is

(a) $x^2 + y^2 - 6x + 4y = 12$
(b) $x^2 + y^2 - 3x + 2y = 12$
(c) $x^2 + y^2 - 6x + 2y = 12$
(d) $x^2 + y^2 - 8x + 6y - 18 = 0$
(e) $x^2 + y^2 - 8x + 6y - 7 = 0$

21. The equation of the parabola with vertex $(-6, 2)$, passing through $(-3, 5)$ and having axis parallel to x-axis is

(a) $(y + 2)^2 = 3x + 16$
(b) $(x + 6)^2 = 3y - 6$
(c) $(y + 2)^2 = 4x + 48$
(d) $(x - 6)^2 = 4y - 8$
(e) $(y - 2)^2 = 3x + 18$

22. One of the vertices of the major axis of an ellipse is $(1, 1)$ and one of the vertices of its minor axis is $(-2, -1)$. If the centre of the ellipse is $(-2, 1)$, then the equation of the ellipse is

(a) $\frac{(x + 2)^2}{9} + \frac{(y - 1)^2}{4} = 1$
(b) $\frac{(x + 2)^2}{16} + \frac{(y - 1)^2}{4} = 1$
(c) $\frac{(x - 2)^2}{9} + \frac{(y + 1)^2}{4} = 1$
(d) $\frac{(x - 2)^2}{16} + \frac{(y + 1)^2}{4} = 1$
(e) $\frac{(x + 2)^2}{9} + \frac{(y - 1)^2}{2} = 1$

23. The equation of the parabola with focus $(3, 0)$ and directrix $x + 3 = 0$ is

(a) $y^2 = 3x - 9$ (b) $y^2 = 4x - 12$
(c) $y^2 = 12x$ (d) $y^2 = 12x - 36$
(e) $y^2 = 12x - 9$

24. The eccentricity of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ is

(a) $\frac{\sqrt{5}}{3}$ (b) $\frac{\sqrt{5}}{6}$ (c) $\frac{\sqrt{30}}{6}$ (d) $\frac{\sqrt{10}}{6}$
(e) $\frac{\sqrt{30}}{7}$

25. The foci of a hyperbola are (8, 3) and (0,3) and eccentricity is $\frac{4}{3}$. Then the length of the transverse axis is

(a) $\frac{32}{3}$ (b) 4 (c) 8 (d) $\frac{8}{3}$
(e) 6

26. The co-ordinates of the points P and Q are (2, 6, 4) and (8, – 3, 1), respectively. If the point R lies on the line segment PQ such that $2|\mathbf{PR}| = |\mathbf{RQ}|$, then the co-ordinates of R are

(a) (4,– 3, 3) (b) (4, 3, – 3)
(c) (2, – 3, 1) (d) (4, 3, 3)
(e) (2, 3, 3)

27. If $|\mathbf{a}| = 2$, $\mathbf{b} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{4}$, then $\mathbf{a} \cdot \mathbf{b}$ is equal to

(a) $14\sqrt{2}$ (b) $2\sqrt{7}$ (c) $\sqrt{30}$ (d) $\sqrt{7}$
(e) $\sqrt{14}$

28. If α is the angle made by the vector $\mathbf{a} = 5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ with the positive x-axis, then $\cos\alpha =$

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{\sqrt{5}}{5}$
(e) $\frac{\sqrt{2}}{10}$

29. If $|\mathbf{a}| = 3, |\mathbf{b}| = 4$ and $|\mathbf{a} - \mathbf{b}| = \sqrt{7}$, then $\mathbf{a} \cdot \mathbf{b}$ is equal to

(a) 7 (b) 8 (c) 9 (d) 10
(e) 12

30. If $\mathbf{a} = \hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\mathbf{b} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{a} \cdot \mathbf{b} = -20$, then the value of λ is equal to

(a) 2 (b) – 2 (c) – 4 (d) 4
(e) 5

31. If $\mathbf{a} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \alpha\hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{a} \times \mathbf{b} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \beta\hat{\mathbf{k}}$, then the value of β is equal to

(a) – 2 (b) 2 (c) – 1 (d) 1
(e) –3

32. The values of α so that the vectors $\alpha\hat{\mathbf{i}} + (\alpha - 1)\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $(\alpha + 2)\hat{\mathbf{i}} + \alpha\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ are perpendicular, are

(a) $\frac{3}{2}, -2$ (b) $2, \frac{3}{2}$ (c) $-2, \frac{-3}{2}$ (d) $2, \frac{-3}{2}$
(e) $-4, \frac{3}{2}$

33. If $|\mathbf{u}| = 5, |\mathbf{v}| = 4$ and the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{6}$, then $|\mathbf{u} \times \mathbf{v}|$ is equal to

(a) $10\sqrt{3}$ (b) $10\sqrt{2}$ (c) 20 (d) $5\sqrt{2}$
(e) 10

34. If the point $P(x, 1, 4)$ lies on the line $\mathbf{r} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}} + \lambda(2\hat{\mathbf{i}} - \hat{\mathbf{j}})$, then the value of x is equal to

(a) 2 (b) – 2 (c) 3 (d) – 3
(e) 5

35. The equation of the plane through the point (2, 1, 3) and perpendicular to the vector $4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ is

(a) $4x + 5y + 6z = 28$ (b) $2x + y + 3z = 17$
(c) $4x + 5y + 6z = 33$ (d) $8x + 5y + 18z = 21$
(e) $4x + 5y + 6z = 31$

36. The angle between the line $\mathbf{r} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + t(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$ and the plane $2x - 3y - z = 1$ is

(a) $\sin^{-1}\left(\frac{1}{196}\right)$ (b) $\sin^{-1}\left(\frac{1}{14}\right)$
(c) $\cos^{-1}\left(\frac{1}{14}\right)$ (d) $\cos^{-1}\left(\frac{13}{14}\right)$
(e) $\sin^{1}\left(\frac{13}{14}\right)$

37. If the line $\mathbf{r} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + t(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ is parallel to the plane $2x + 4y + az = 8$, then the value of a is equal to

(a) 2 (b) 3 (c) 4 (d) 5
(e) 6

38. The angle between the lines $\mathbf{r} = \hat{\mathbf{i}} + 4\hat{\mathbf{k}} + \lambda(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ and $\mathbf{r} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \mu(3\hat{\mathbf{i}} + \hat{\mathbf{k}})$ is

(a) $\cos^{-1}\left(\frac{\sqrt{5}}{6}\right)$ (b) $\cos^{-1}\left(\frac{\sqrt{15}}{6}\right)$
(c) $\cos^{-1}\left(\frac{1}{12}\right)$ (d) $\cos^{-1}\left(\frac{\sqrt{15}}{15}\right)$
(e) $\cos^{-1}\left(\frac{\sqrt{3}}{30}\right)$

39. The Cartesian equation of the line passing through (7, 5, 3) and perpendicular to the plane $3x + 2y + z = 6$ is

(a) $\frac{x-7}{3} = \frac{y-5}{2} = \frac{z-3}{1}$
(b) $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{3}$
(c) $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z}{3}$
(d) $\frac{x-7}{3} = \frac{y-5}{1} = \frac{z-3}{2}$
(e) $\frac{x-4}{4} = \frac{y-3}{3} = \frac{z-2}{2}$

40. The acute angle between the planes $2x - y - 3z = 7$ and $x + 2y + 2z = 0$ is

(a) $\cos^{-1}\left(\frac{-\sqrt{14}}{14}\right)$ (b) $\pi - \cos^{-1}\left(\frac{-\sqrt{14}}{7}\right)$
(c) $\cos^{-1}\left(\frac{\sqrt{14}}{11}\right)$ (d) $\pi - \cos^{-1}\left(\frac{-\sqrt{14}}{21}\right)$
(e) $\pi - \cos^{-1}\left(\frac{\sqrt{14}}{7}\right)$

41. The vector equation of the line joining the points (2, 1, 3) and $(-2, 4, 1)$ is

(a) $r = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda(-4\hat{i} + 3\hat{j} - 2\hat{k})$
(b) $r = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda(4\hat{i} + 3\hat{j} + 2\hat{k})$
(c) $r = -2\hat{i} + \hat{j} + 3\hat{k} + \lambda(-4\hat{i} - 3\hat{j} - 2\hat{k})$
(d) $r = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda(3\hat{i} - 4\hat{j} - 2\hat{k})$
(e) $r = -4\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$

42. A bag contains 5 yellow, 3 green, 2 blue and 7 white balls. If 4 balls are chosen at random, then the probability that none of them are white is

(a) $\frac{3}{37}$ (b) $\frac{7}{34}$ (c) $\frac{5}{34}$ (d) $\frac{5}{37}$
(e) $\frac{3}{34}$

43. An urn contains 25 marbles which are numbered from 1 to 25 and a marble is chosen at random two times with replacement. Then the probability that both times the marble has the same number is

(a) $\frac{1}{25}$ (b) $\frac{24}{25}$ (c) $\frac{1}{625}$ (d) $\frac{624}{625}$
(e) $\frac{2}{25}$

44. If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.55$ and $P(A \cap B) = 0.1$, then $P(B \cap A^C)$ is equal to

(a) 0.25 (b) 0.35 (c) 0.45 (d) 0.65
(e) 0.75

45. Two dice are rolled. If A is the event that sum of the numbers is 4 and B is the event that at least one of the dice shows a 3, then $P(A|B)$ is equal to

(a) $\frac{3}{11}$ (b) $\frac{2}{11}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$
(e) $\frac{1}{11}$

46. Assume that n distinct values $x_1, x_2, \ldots, x_n$ occur with frequencies $f_1, f_2, \ldots, f_n$ respectively. If $\bar{x} = 7$ and $\sum_{i=1}^{8} f_i x_i = 315$, then $\sum_{i=1}^{8} f_i =$

(a) 35 (b) 45 (c) 48 (d) 42
(e) 40

47. The variance of the data $x_1, x_2, \ldots, x_{50}$ with $\sum_{i=1}^{50} x_i = 650$ and $\sum_{i=1}^{50} x_i^2 = 10000$ is

(a) 30 (b) 40 (c) 39 (d) 41
(e) 31

48. If X is a random variable with $E(X) = 6$ and $V(X) = 3$, then $E(X^2)$ is equal to

(a) 33 (b) 36 (c) 39 (d) 42
(e) 27

49. Let $f(x) = \frac{4x+3}{x+2}$. Then the value of $f^{-1}(-2)$ is equal to

(a) $\frac{7}{5}$ (b) $\frac{-7}{6}$ (c) $\frac{-7}{5}$ (d) $\frac{7}{6}$
(e) $\frac{5}{6}$

50. If $f(x) = \begin{cases} 2x & , x < 1 \\ 5a - x & , x \geq 1 \end{cases}$ is continuous on R, then the value of a is equal to

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$
(e) 1

51. $\lim_{t\to 0}\frac{\sin 2t}{8t^2+4t}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
(e) 1

52. $\lim_{x\to 0}\frac{x}{\sqrt{9-x}-3}$ is equal to

(a) 6 (b) 3 (c) – 3 (d) – 6
(e) 0

53. Let $f(x)=\begin{cases}3x+2, & \text{if } x<-2\\ x^2-3x-1, & \text{if } x\geq -2\end{cases}$. Then $\lim_{x\to -2^-} f(x)$ and $\lim_{x\to -2^+} f(x)$ are respectively

(a) –4, 3 (b) 6, 3 (c) –6, 3 (d) –4, 9
(e) 9, –4

54. $\lim_{x\to -3}\frac{x^2+16x+39}{2x^2+7x+3}$ is equal to

(a) 2 (b) $\frac{8}{3}$ (c) $\frac{-8}{3}$ (d) –2
(e) 0

55. Let $f(x)=6\sqrt[3]{x^5}$. If $f'(x)=ax^p$, where a and p are constants, then the value of p is equal to

(a) $\frac{3}{5}$ (b) $\frac{-2}{5}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$
(e) $\frac{2}{5}$

56. Let $y=(\tan x)^{\sin x}$ for $0<x<\frac{\pi}{2}$. If $\frac{dy}{dx}=(\tan x)^{\sin x}((\cos x)\log(\tan x)+g(x))$, then $g(x)=$

(a) $\sin x\sec^2 x$ (b) $\sec x\operatorname{cosec} x$
(c) $\sec x$ (d) cosec x
(e) $\sin x\tan x$

57. If $f(x)=(x^3+\sin\pi x)^5$, then $f'(1)$ is equal to

(a) 2^5 (b) $5(2^4)$ (c) 15 (d) $5(3+\pi)$
(e) $5(3-\pi)$

58. If $h(x)=4x^3-5x+7$ is the derivative of $f(x)$, then $\lim_{t\to 0}\frac{f(1+t)-f(1)}{t}$ is equal to

(a) 5 (b) 6 (c) 7 (d) 8
(e) 0

59. Let $f(x)=\begin{cases}e^x, & \text{if } x\leq 1\\ mx+6, & \text{if } x>1\end{cases}$ be differentiable at $x=1$. Then the value of m is

(a) 6 (b) e (c) –6 (d) $-e$
(e) 1

60. $\lim_{t\to 0}\frac{\tan^2\left(\frac{\pi}{3}+t\right)-3}{t}$ is equal to

(a) $4\sqrt{3}$ (b) 24 (c) $16\sqrt{3}$ (d) $8\sqrt{3}$
(e) 16

61. If the tangent line to the graph of a function f at the point $x=3$ has x-intercept $\frac{5}{3}$ and y-intercept -10, then $f'(3)$ is equal to

(a) 3 (b) 5
(c) $\frac{5}{3}$ (d) 6
(e) – 10

62. The slope of tangent line to the curve $4x^2+2xy+y^2=12$ at the point $(1, 2)$ is

(a) 2 (b) 1 (c) –1 (d) –2
(e) 0

63. Let $f(x)=\sqrt{x}+5$ for $1\leq x\leq 9$. Then the value of c whose existence is guaranteed by the Mean Value Theorem is

(a) 2 (b) 3 (c) 4 (d) 5
(e) 6

64. The derivative of a function f is given by $f'(x)=\frac{x-5}{\sqrt{x^2+4}}$. Then the interval in which f is increasing, is

(a) $(5, \infty)$ (b) $(0, \infty)$
(c) $(-4, \infty)$ (d) $(-\infty, -4)$
(e) $(-\infty, 5)$

65. Let $f(x)+x^2\log x$, $x>0$. Then the minimum value of f is

(a) $\frac{1}{\sqrt{e}}$ (b) $2e$
(c) $-2e$ (d) $\sqrt{e}$
(e) $\frac{-1}{2e}$

66. A cube is expanding in such a way that its edge is increasing at a rate of 2 inches/sec. If its edge is 5 inches long, then the rate of change of its volume is

(a) 150 inches^3/sec (b) 75 inches^3/sec
(c) 50 inches^3/sec (d) 30 inches^3/sec
(e) 45 inches^3/sec

67. $\int x^5 e^{1-x^6} dx =$

(a) $\frac{1}{6}e^{1-x^6} + C$ (b) $-e^{1-x^6} + C$
(c) $\frac{-1}{6}e^{1-x^6} + C$ (d) $\frac{x^5}{5}e^{1-x^6} + C$
(e) $\frac{x^6}{6}e^{1-x^6} + C$

68. $\int (5-4x)e^{-x} dx =$

(a) $e^{-x}(4x-1)+C$ (b) $e^{-x}(9-4x)+C$
(c) $e^{-x}(4x-5)+C$ (d) $e^{-x}(4x-9)+C$
(e) $e^{-x}(5-4x)+C$

69. $\int \frac{\cos(\tan x)}{\cos^2 x} dx =$

(a) $(\tan x)\sin(\tan x)+C$ (b) $\sin(\tan x)+C$
(c) $\sec(\tan x)+C$ (d) $(\cos x)\sin(\tan x)+C$
(e) $\cos^2(\tan x)+C$

70. $\int \frac{1}{e^{2x}-1} dx =$

(a) $2\log|e^{2x}-1| - x + C$ (b) $x - \frac{1}{2}\log|e^{2x}-1| + C$
(c) $x + \frac{1}{2}\log|e^{2x}-1| + C$ (d) $x - \log|e^{2x}-1| + C$
(e) $\frac{1}{2}\log|e^{2x}-1| - x + C$

71. $\int \sin 2x \cos x \, dx =$

(a) $\frac{-1}{3}\cos^3 x + C$ (b) $\frac{-2}{3}\cos^3 x + C$
(c) $\frac{2}{3}\cos^3 x + C$ (d) $\frac{1}{3}\cos^3 x + C$
(e) $\frac{-4}{3}\cos^3 x + C$

72. $\int \frac{1}{(1+\cot^2 x)\sin^2 x} dx =$

(a) $\tan^{-1}(\sin x)+C$ (b) $\tan^{-1}(\cos x)+C$
(c) $\cot^{-1}(\sin x)+C$ (d) $\cot^{-1}(\cos x)+C$
(e) $x+C$

73. $\int \frac{4x^9}{x^{10}-10} dx =$

(a) $\frac{1}{5}\log|x^{10}-10|+C$
(b) $\frac{2}{5}\log|x^{10}-10|+C$
(c) $\frac{1}{10}\log|x^{10}-10|+C$
(d) $\frac{-2}{5}\log|x^{10}-10|+C$
(e) $\frac{-1}{10}\log|x^{10}-10|+C$

74. The value of $\int_0^{\sqrt{3}} \frac{6}{9+x^2} dx$ is equal to

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$
(e) 1

75. The value of $\int_{-5}^{5} (4-|x|) dx$ is equal to

(a) 18 (b) 10 (c) 12 (d) 16
(e) 15

76. The area of the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$ is (in square units)

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$
(e) 1

77. The value of $\int_0^2 \frac{x^2}{(x^3+1)^2} dx$ is equal to

(a) $\frac{1}{27}$ (b) $\frac{5}{27}$ (c) $\frac{7}{27}$ (d) $\frac{8}{27}$
(e) $\frac{1}{3}$

78. The value of $\int_{\pi/8}^{3\pi/8} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$
(c) $\frac{\pi}{16}$ (d) $\frac{\pi}{2}$
(e) 1

79. The area of the region bounded by $y = 5x$, x-axis and $x = 4$ is (in square units)

(a) 40 (b) 80
(c) 20 (d) 50
(e) 60

80. The general solution of the differential equation $y - xy' = x^2 + y^2$ is

(a) $y = x\tan(C - x)$ (b) $y = \tan x + C$
(c) $y = x^2 \tan x + C$ (d) $y = x\tan x + C$
(e) $y = x\tan x + Cx$

81. The integrating factor of the differential equation $xy' + 2y - 7x^3 = 0$ is

(a) $\log|x|$ (b) x^2 (c) $\frac{1}{x^2}$ (d) $\frac{1}{2}\log|x|$
(e) x

82. The general solution of the differential equation $4xy + 12x + (2x^2 + 3)y' = 0$ is

(a) $\frac{2x^2 + 3}{y + 3} = C$ (b) $\frac{y - 3}{2x^2 + 3} = C$
(c) $\frac{y + 2}{2x^2 + 3} = C$ (d) $(y - 3)(2x^2 + 3) = C$
(e) $(y + 3)(2x^2 + 3) = C$

83. The constraints of a linear programming problem are $x + 2y \le 10$ and $6x + 3y \le 18$. Which of the following points lie in the feasible region?

(a) (0, 6) (b) (4, 3) (c) (5, 7) (d) (1, 7)
(e) (1, 3)

84. Let $f : [-4, 2] \to R$ be given by $f(x) = \sqrt{16 - x^2}$. Then the range of the function f is

(a) [0, 2] (b) $[0, 2\sqrt{3}]$
(c) [0, 4] (d) $[2\sqrt{3}, 4]$
(e) [–2,2]

85. Let $f(x) = x^2$ and $g(x) = \sqrt{9 + x}$. Then the value of $(fog - gof)(4)$ is equal to

(a) 6 (b) $\sqrt{6}$ (c) $\sqrt{8}$ (d) 8
(e) 5

86. Let A and B be subsets of the universal set U. If $n(A) = 24$, $n(A \cap B) = 8$ and $n(U) = 63$, then $n(A' \cup B')$ is equal to

(a) 43 (b) 55 (c) 35 (d) 32
(e) 45

87. Let $f(x) = [x]$, $x \in R$, where $[x]$ denotes the greatest integer $\le x$. Then the images of the elements – 4.6 and 2.7 are respectively

(a) –5, 2 (b) –5, 3 (c) – 4, 2 (d) – 3, 3
(e) – 4, 3

88. For any two positive rational numbers m and n, a binary operation * is defined by $m * n = \frac{m + n}{3}$, then $\frac{7}{2} * \frac{5}{2}$ is equal to

(a) 4 (b) 6 (c) 2 (d) 8
(e) 9

89. The function $f : R \to R$ given $f(x) = 7 - 3x$ is

(a) not one-one (b) not onto
(c) even (d) one-one and onto
(e) odd

90. A relation R on {0, 1, 2} is given by R = {(0, 0), (1, 1), (0, 1), (2, 2), (1, 2)}. Then the relation R is

(a) reflexive
(b) symmetric
(c) transitive
(d) symmetric and transitive
(e) equivalence

91. Let z_1, z_2 and z_3 be three distinct points in the complex plane such that the segment joining z_1 and z_2 is perpendicular to the segment joining z_1 and z_3. If $|z_1 - z_2| = 5$ and $|z_1 - z_3| = 12$, then $|z_2 - z_3|$ is equal to

(a) 17 (b) 7 (c) 13 (d) 14
(e) 9

92. If $\frac{Z}{i} = 11 - 13i$, then $z + \bar{z}$ is equal to

(a) –22 (b) 22 (c) 25 (d) 26
(e) –26

93. Let $\alpha = 2 - 3i$ be a root of the equation $z^2 - 4z + k = 0$, where k is a real number. If β is the other root, then the value of $\alpha^2 + \beta^2$ is

(a) 26 (b) – 5 (c) 5 (d) 10
(e) – 10

94. If $z = 2 - i\sqrt{3}$, then $|z^4|$ is equal to

(a) 7 (b) $\sqrt{7}$
(c) $7\sqrt{7}$ (d) 49
(e) $49\sqrt{7}$

95. The imaginary part of $z = \frac{2 + i}{3 - i}$ is

(a) $\frac{5}{8}$ (b) $\frac{-5}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$
(e) $\frac{3}{8}$

96. The area of the triangle on the complex plane formed by the points $z, z+iz$ and iz is 128. Then the value of $|z|$ is

(a) 12 (b) 16 (c) 18 (d) 17
(e) 19

97. If the real part of the complex number $z = \dfrac{p+2i}{p-i}$, $p \in R$, $p > 0$ is $\dfrac{1}{2}$, then the value of p is equal to

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\dfrac{\sqrt{3}}{2}$
(e) 1

98. The value of $\sqrt{(-25)} + 3\sqrt{(-4)} + 2\sqrt{(-9)}$ is equal to

(a) $13i$ (b) $-13i$ (c) $11i$ (d) $-17i$
(e) $17i$

99. The value of $\sum_{k=5}^{36} \dfrac{1}{k^2 - k}$ is

(a) $\dfrac{7}{36}$ (b) $\dfrac{1}{9}$ (c) $\dfrac{2}{9}$ (d) $\dfrac{1}{12}$
(e) $\dfrac{5}{36}$

100. If $a_1, a_2, a_3, \ldots, a_n$ are in AP with $a_1 = 3$, $a_n = 39$ and $a_1 + a_2 + \ldots + a_n = 210$, then the value of n is equal to

(a) 8 (b) 10 (c) 11 (d) 13
(e) 15

101. Let t_n, $n = 1, 2, 3, \ldots$ be the n^{th} term of the AP 5, 8, 11, … . Then the value of n for which $t_n = 305$ is

(a) 101 (b) 100 (c) 103 (d) 99
(e) 95

102. If the first term of a GP is 1 and the sum of 3rd and 5th terms is 90, then the positive common ratio of the GP is

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

103. In an AP, the difference between the last and the first term is 632 and the common difference is 4. Then the number of terms in the AP is

(a) 157 (b) 160
(c) 158 (d) 159
(e) 140

104. If the 10th and 12th terms of an A.P. are respectively 15 and 21, then the common difference of the AP is

(a) -6 (b) 4 (c) 6 (d) -3
(e) 3

105. The first term of a GP is 3 and the common ratio is 2. Then the sum of first eight terms of the GP is

(a) 763 (b) 189 (c) 381 (d) 765
(e) 655

106. A covid-19 vaccination reduces the probability of getting covid-19 infection from 0.4 to 0.1. In a city, 45% people are vaccinated. Then the probability that a non-vaccinated person chosen at random in the city gets covid-19 infection is

(a) 0.55 (b) 0.45 (c) 0.32 (d) 0.22
(e) 0.18

107. The number of ways of a committee of 3 women and 5 men can be formed from a panel of 8 men and 5 women is

(a) 940 (b) 1120 (c) 560 (d) 760
(e) 520

108. A set contains 9 elements. Then the number of subsets of the set which contains at most 4 elements is

(a) 32 (b) 64 (c) 128 (d) 256
(e) 512

109. If p and q are positive integers, such that ${}^{(p+q)}P_2 = 42$ and ${}^{(p-q)}P_2 = 20$, then the values of p and q are respectively

(a) 5, 2 (b) 4, 3
(c) 7, 2 (d) 6, 1
(e) 7, 5

110. The number of 3-digits numbers that can be formed from the digits 0, 2, 3, 5, 7 is (repetition is allowed)

(a) 125 (b) 100 (c) 105 (d) 150
(e) 60

111. If x^{22} is in the $(r+1)$th term of the binomial expansion of $(3x^3 - x^2)^9$, then the value of r is equal to

(a) 3 (b) 4 (c) 5 (d) 6
(e) 7

112. The term independent of x in the binomial expansion of $\left(x+\frac{2}{x^3}\right)^{20}$ is

(a) $\binom{20}{5}2^{15}$ (b) $\binom{20}{15}2^{10}$ (c) $\binom{20}{10}2^{5}$ (d) $\binom{20}{10}2^{10}$
(e) $\binom{20}{5}2^{5}$

113. Let $A+B=\begin{bmatrix}4 & 1 & 4\\1 & 4 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & -2\\-1 & 3 & 0\end{bmatrix}$, then $A=$

(a) $\begin{bmatrix}3 & 1 & 2\\0 & 3 & 4\end{bmatrix}$ (b) $\begin{bmatrix}5 & 1 & 2\\0 & 7 & 4\end{bmatrix}$
(c) $\begin{bmatrix}3 & -1 & -2\\2 & 1 & 4\end{bmatrix}$ (d) $\begin{bmatrix}5 & 1 & 6\\2 & 1 & 4\end{bmatrix}$
(e) $\begin{bmatrix}3 & 1 & 6\\2 & 1 & 4\end{bmatrix}$

114. The value of the determinant $\begin{vmatrix}4 & 4^2 & 4^3\\3 & 3^2 & 3^3\\2 & 2^2 & 2^3\end{vmatrix}$ is

(a) 52 (b) – 24 (c) 24 (d) 48
(e) – 48

115. If, $\begin{vmatrix}1 & 2 & 1\\0 & x & -3\\2 & -1 & x\end{vmatrix}=0$, then the values of x are

(a) 5, – 3 (b) 5, 3 (c) – 5, 3 (d) 2, 3
(e) – 2, – 3

116. If $AB=\begin{bmatrix}4 & 3\\5 & 4\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}3 & -2\\-1 & 1\end{bmatrix}$, then $B=$

(a) $\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 2\\1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}$
(e) $\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}$

117. The matrix $\begin{bmatrix}-2 & 1 & 0\\3 & 4 & 1\\-4 & \lambda & 0\end{bmatrix}$ is non-singular for $\lambda \neq$

(a) 2 (b) –2
(c) 4 (d) – 4
(e) 0

118. Let $\begin{vmatrix}x-1 & 2 & 1\\2 & x-1 & 2\\1 & x+2 & x-1\end{vmatrix}=ax^3+bx^2+cx+d$, where a, b, c and d are constants. Then the value of d is

(a) – 8 (b) 6
(c) 0 (d) – 6
(e) 16

119. If the inequality $-13 \leq x \leq 5$ is expressed in the form $|x-a| \leq b$, then the values of a and b are respectively

(a) 4, 8 (b) –4, 9
(c) 4, 9 (d) 5, 9
(e) –5, 9

120. The solution set of the inequality $5(4x+6)<25x+10$ is

(a) $(4, \infty)$ (b) $(-\infty, 4)$
(c) $(-\infty, 5)$ (d) $(5, \infty)$
(e) $(-4, 4)$

Answers

Physics & Chemistry

1.	(c)	2.	(d)	3.	(b)	4.	(a)	5.	(b)	6.	(a)	7.	(e)	8.	(d)	9.	(a)	10.	(c)
11.	(e)	12.	(b)	13.	(d)	14.	(a)	15.	(b)	16.	(c)	17.	(b)	18.	(c)	19.	(d)	20.	(e)
21.	(d)	22.	(e)	23.	(c)	24.	(d)	25.	(b)	26.	(c)	27.	(e)	28.	(d)	29.	(*)	30.	(d)
31.	(d)	32.	(e)	33.	(b)	34.	(a)	35.	(e)	36.	(c)	37.	(e)	38.	(b)	39.	(a)	40.	(a)
41.	(c)	42.	(b)	43.	(c)	44.	(a)	45.	(d)	46.	(a)	47.	(d)	48.	(c)	49.	(b)	50.	(b)
51.	(c)	52.	(b)	53.	(c)	54.	(d)	55.	(b)	56.	(d)	57.	(c)	58.	(c)	59.	(d)	60.	(e)
61.	(e)	62.	(c)	63.	(c)	64.	(d)	65.	(a)	66.	(b)	67.	(a)	68.	(*)	69.	(a)	70.	(d)
71.	(e)	72.	(a)	73.	(d)	74.	(a)	75.	(c)	76.	(b)	77.	(e)	78.	(d)	79.	(b)	80.	(a)
81.	(c)	82.	(a)	83.	(b)	84.	(c)	85.	(d)	86.	(a)	87.	(b)	88.	(a)	89.	(b)	90.	(b)
91.	(e)	92.	(a)	93.	(b)	94.	(e)	95.	(c)	96.	(e)	97.	(e)	98.	(c)	99.	(d)	100.	(a)
101.	(d)	102.	(e)	103.	(d)	104.	(a)	105.	(c)	106.	(e)	107.	(c)	108.	(d)	109.	(b)	110.	(e)
111.	(e)	112.	(d)	113.	(a)	114.	(b)	115.	(c)	116.	(c)	117.	(b)	118.	(a)	119.	(d)	120.	(c)

Mathematics

1.	(c)	2.	(d)	3.	(a)	4.	(b)	5.	(c)	6.	(d)	7.	(d)	8.	(b)	9.	(c)	10.	(a)
11.	(e)	12.	(c)	13.	(d)	14.	(a)	15.	(e)	16.	(a)	17.	(a)	18.	(b)	19.	(c)	20.	(a)
21.	(e)	22.	(a)	23.	(c)	24.	(a)	25.	(e)	26.	(d)	27.	(b)	28.	(c)	29.	(c)	30.	(d)
31.	(d)	32.	(a)	33.	(e)	34.	(e)	35.	(e)	36.	(b)	37.	(d)	38.	(b)	39.	(a)	40.	(b)
41.	(a)	42.	(e)	43.	(a)	44.	(c)	45.	(b)	46.	(b)	47.	(e)	48.	(c)	49.	(b)	50.	(c)
51.	(a)	52.	(d)	53.	(d)	54.	(d)	55.	(c)	56.	(c)	57.	(e)	58.	(b)	59.	(b)	60.	(d)
61.	(d)	62.	(d)	63.	(c)	64.	(a)	65.	(e)	66.	(a)	67.	(c)	68.	(a)	69.	(b)	70.	(e)
71.	(b)	72.	(e)	73.	(b)	74.	(a)	75.	(e)	76.	(b)	77.	(d)	78.	(b)	79.	(a)	80.	(a)
81.	(b)	82.	(e)	83.	(e)	84.	(c)	85.	(d)	86.	(b)	87.	(a)	88.	(c)	89.	(d)	90.	(a)
91.	(c)	92.	(d)	93.	(e)	94.	(d)	95.	(c)	96.	(b)	97.	(b)	98.	(e)	99.	(c)	100.	(b)
101.	(a)	102.	(c)	103.	(d)	104.	(e)	105.	(d)	106.	(d)	107.	(c)	108.	(d)	109.	(d)	110.	(b)
111.	(c)	112.	(e)	113.	(e)	114.	(e)	115.	(a)	116.	(e)	117.	(a)	118.	(e)	119.	(b)	120.	(a)

(*) None of the option is correct.

Answer *with* Explanations

Physics

1. *(c)* When two sound waves of slightly different frequencies f_1 and f_2 are sounded together, they superimpose at a given point, which give beats.

The time interval between the successive maxima is expressed as

$$T = \frac{1}{f_1 - f_2}$$

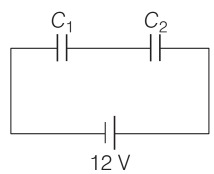
2. *(d)* Electrostatic potential at a point at a distance r due to an electric dipole is given as

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2}$$

where, p is dipole moment.

$$\Rightarrow \quad V \propto \frac{1}{r^2}$$

3. *(b)* Let the air filled capacitor and identical capacitor with dielectric medium having capacitance C_1 and C_2 is as shown in the figure below



Given, dielectric constant, $K = 5$

Since, the capacitors are connected in series, so charge on each capacitor remains same, while potential difference is distributed inversely as the ratio of capacitance, i.e.

$$V_1 : V_2 = \frac{1}{C_1} : \frac{1}{C_2}$$

As we know, for a parallel plate capacitor, capacitance, $C = \frac{\varepsilon_0 A}{d}$, where A is the area of plates of capacitor and d is the distance between the plates.

$\therefore$ For air filled capacitor, $C_1 = \frac{\varepsilon_0 A}{d}$ ... (i)

Similarly, $C_2 = \frac{K\varepsilon_0 A}{d}$ ... (ii)

($\because$ the capacitors are identical)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{C_1}{C_2} = \frac{\varepsilon_0 A}{d} \Big/ \frac{K\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d} \times \frac{d}{K\varepsilon_0 A} = \frac{1}{K}$$

or $C_1 : C_2 = 1 : K = 1 : 5$

$\Rightarrow$ $V_1 : V_2 = \frac{1}{C_1} : \frac{1}{C_2} = 5 : 1$

or $\frac{V_1}{V_2} = \frac{5}{1} \Rightarrow V_1 = 5V_2$ ... (iii)

As, $V_1 + V_2 = 12$ (given)

$5V_2 + V_2 = 12$ [from Eq. (iii)]

or $6V_2 = 12$

$\Rightarrow$ $V_2 = 2\text{V}$

and $V_1 = 5V_2 = 5 \times 2 = 10\text{ V}$

4. *(a)* Electrostatic force between two charges q_1 and q_2 is given as

$$F = \frac{Kq_1q_2}{r^2}$$

where, K is the Coulomb's constant and r is the distance between the charges.

According to the question, for two protons with charge q (say) and at a distance r apart, the electrostatic force is given as

$$F_p = \frac{Kqq}{r^2} = \frac{Kq^2}{r^2} \quad \text{... (i)}$$

Similarly, for two electrons with charge e and at a distance r apart, the electrostatic force is given as

$$F_e = \frac{Kee}{r^2} = \frac{Ke^2}{r^2} \quad \text{... (ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{F_p}{F_e} = \frac{Kq^2}{r^2} \Big/ \frac{Ke^2}{r^2}$$

$$\Rightarrow \quad \frac{F_p}{F_e} = \frac{q^2}{e^2}$$

Since, the magnitude of charge on proton = the magnitude of charge on electron

i.e. $|q| = |e| = 1.6 \times 10^{-19}\text{ C}$

$$\Rightarrow \quad \left|\frac{F_p}{F_e}\right| = \left|\frac{q^2}{e^2}\right| = \left|\frac{q^2}{q^2}\right| = \frac{1}{1}$$

or $\quad |F_p : F_e| = 1:1$

5. *(b)* According to Coulomb's law, force between two charges at a certain distance d apart in air medium is given as,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2} \quad \ldots \text{(i)}$$

where, ε_0 is the permittivity of free space.

When these charges are kept in a dielectric medium, then force between them is given as

$$F_D = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{d^2} \quad \ldots \text{(ii)}$$

where, K is the dielectric constant.

Given, $\quad F_D = \frac{F}{2}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{2d^2}$$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{d^2}$$

[from Eqs. (i) and (ii)]

$$\Rightarrow \quad K = 2$$

6. *(a)* **Mobility** is defined as drift velocity per unit electric field, i.e.

$$\mu = \frac{v_d}{E}$$

However, **current density** is defined as current per unit area.

Resistivity is defined as the electrical resistance of a conductor of a unit cross-sectional area and unit length.

Conductivity is the reciprocal of resistivity.

7. *(e)* In balanced condition, in a Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

Given, $P = 5\Omega$, $Q = 10\Omega$, $R = 20\Omega$, $S = 40\Omega$

If 40Ω resistor is connected across S arm, then net resistance offered by this arm is

$$S_{\text{net}} = \frac{40 \times 40}{40 + 40} = 20\ \Omega$$

Now, in order to balance the bridge again, 10Ω resistor has to connected across Q, so the net resistance offered by Q_{net} will be 5Ω.

So, the new balanced condition will be

$$\frac{P}{Q_{\text{net}}} = \frac{R}{S_{\text{net}}}$$

$$\Rightarrow \quad \frac{5}{5} = \frac{20}{20}$$

8. *(d)* Given, emf of each cell, $\varepsilon = 1.5\,\text{V}$

Internal resistance of each cell, $r = 0.5\ \Omega$

Number of identical cells, $n = 4$

Equivalent internal resistance, $r_{\text{eq}} = nr$

$$r_{\text{eq}} = 4 \times 0.5 = 2\ \Omega$$

9. *(a)* According to the colour coding of the carbon resistors,

Blue coloured strip corresponds to 6.

Black coloured strip corresponds to 0.

Multiplier coloured strip corresponding to 10^2.

Silver coloured strip corresponds to tolerance value of 10%.

$\therefore$ Resistance in ohm $= 60 \times 10^2 \pm 10\%$

10. *(c)* Given, length of conductor,

$l = 20\text{ cm} = 0.2\text{ m}$

Current, $i = 5\text{A}$, $\quad \theta = 30°$

Magnetic field, $\quad B = 0.5\text{ T}$

Force acting on the conductor in the external magnetic field is given as

$$\mathbf{F} = i\,\mathbf{l} \times \mathbf{B}$$

In terms of magnitude,

$$F = ilB\sin\theta$$

Substituting the given values, we get

$$F = 5 \times 0.2 \times 0.5 \times \sin 30°$$

$$= 5 \times 0.2 \times 0.5 \times \frac{1}{2} = 0.25\,\text{N}$$

11. *(e)* When a current carrying coil is placed in magnetic field B experiences a torque which is given as

$$\tau = \mathbf{M} \times \mathbf{B}$$

$$= MB\sin\theta$$

where, M is the magnetic moment.

So, τ is maximum at $\theta = 90°$,

i.e. $\quad \tau_{\max} = MB$

τ is minimum at $\quad \theta = 0$,

i.e. $\quad \tau_{\min} = 0$

Now, the flux associated with the coil will be

$$\phi = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$

So, ϕ is maximum at $\theta = 0$

$\Rightarrow \quad \phi_{max} = BA$

ϕ is minimum at $\theta = 90°$

$\Rightarrow \quad \phi_{min} = 0$

12. *(b)* Cyclotron frequency is given as

$$f = \frac{Bq}{2\pi m}$$

where, B is magnetic field,

q is charge on the particle

and m is the mass of the charged particle.

Thus, f is independent of the energy of the charged particle.

13. *(d)* Hard ferromagnetic materials are materials that can maintain constant magnetic properties once they are magnetised. They are used for making permanent magnets.

e.g. Alnico, alcomax, ferrites, etc.

14. *(a)* At a point situated at a distance r from the centre of a current carrying circular coil along the axial line, the magnetic field is given as

$$B = \frac{\mu_0 i R^2}{2(R^2 + r^2)^{3/2}}$$

where, i is the current in the coil

and R is the radius of the coil.

Given, $\quad r = R$

$$\Rightarrow \quad B = \frac{\mu_0 i R^2}{2(R^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{2 \cdot 2^{3/2} R} \quad \ldots \text{(i)}$$

Since, magnetic field due to a current carrying circular coil at centre is given as

$$B = \frac{\mu_0 i}{2R} = B_c \quad \text{(given)} \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$B = \frac{B_c}{2\sqrt{2}}$$

15. *(b)* Inductance of a solenoid coil of length L, total number of turns N and cross-sectional area A, is given as

$$L = \frac{\mu_0 N^2 A}{l} \quad \ldots \text{(i)}$$

where, μ_0 is the permeability in vacuum (or air).

If air core is replaced by iron core, inductance will be given as

$$L' = \frac{\mu N^2 A}{l} \quad \ldots \text{(ii)}$$

On dividing Eq. (ii) by Eq (i). we get

$$\frac{L'}{L} = \frac{\mu N^2 A}{l} / \frac{\mu_0 N^2 A}{l}$$

$$= \frac{\mu}{\mu_0} = \mu_r$$

where, μ_r is the relative permeability of iron.

Given, $L = 0.02$ mH, $L' = 40$ mH

$$\because \quad \mu_r = \frac{40}{0.02} = 2000$$

16. *(c)* Average power dissipated in an AC circuit is given as

$$P_{av} = V_{rms} i_{rms} \cos\phi$$

where, V_{rms} and i_{rms} are the rms value of voltage and current respectively and $\cos\phi$ is the power factor.

The circuit gives maximum power dissipation, i.e. the voltage and current are in same phase, so phase difference is zero, or power factor is maximum, which occurs only in resistive circuit.

So, the circuit that gives maximum power dissipation is purely resistive circuit.

17. *(b)* When a changing magnetic flux is applied to a bulk piece of conducting material, then circulating current induced in the body of the conductor are called eddy currents.

They are useful in electromagnetic damping, induction furnace, electric power meters, magnetic brakes in electronic trains.

18. *(c)* At the magnetic equator, angle of dip (θ) is $0°$.

As, $\quad H = B_e \cos\theta$

where, B_e is the net magnetic field and H is the horizontal component of B_e.

$\Rightarrow \quad H = B_e \cos 0° = B_e$

$\quad V = B_e \sin 0° = 0$

Total intensity of magnetic field,

$$B = \sqrt{H^2 + V^2} = B_e \quad \ldots \text{(i)}$$

However, θ at poles is $90°$.

$\Rightarrow \quad H = B_e \cos 90° = 0$

$\quad V = B_e \sin 90° = B_e$

$\Rightarrow$ Total intensity of magnetic field,

$$B = \sqrt{H^2 + V^2} = B_e$$

Given, $\quad B_e = 7 \quad \ldots \text{(ii)}$

From Eqs. (i) and (ii), we get

Total intensity of earth's magnetic field at equator = 7 units

19. *(d)*

EM waves	Detection devices
Microwaves	Point contact diode
Gamma rays	Ionisation chamber
X-rays	Photographic film
Ultraviolet rays	Photocells, photographic film
Infrared rays	Thermopiles, bolometer

20. *(e)* Electromagnetic waves are the waves in which electric and magnetic field vectors change sinusoidally and are perpendicular to each other and as well as to the direction of propagation of wave $(\mathbf{E} \times \mathbf{B})$.

Also, $\mathbf{E}$ and $\mathbf{B}$ are in phase with each other.

21. *(d)* When a slit or hole size a is illuminated by a parallel beam, then it is diffracted at an angle of $\frac{\lambda}{a}$.

In travelling a distance z, size of beam is $\frac{z\lambda}{a}$.

So, $$\frac{z\lambda}{a} \geq a$$

or $$z \geq \frac{a^2}{\lambda}$$

where, z is Fresnel's distance. So, image formation can be explained by ray optics for distance less than z_f.

22. *(e)* Given, apparent depth $(d_{\text{app}})_{\text{water}} = 9\text{ cm}$

$$\mu_{\text{water}} = 4/3$$
$$\mu_{\text{liquid}} = 1.5$$

As we know,

$$d_{\text{app}} = \frac{\text{actual depth}}{\text{refractive index }(\mu)}$$

$$\Rightarrow \quad d_{\text{app}} \propto \frac{1}{\mu}$$

$$\Rightarrow \quad \frac{(d_{\text{app}})_{\text{liquid}}}{(d_{\text{app}})_{\text{water}}} = \frac{\mu_{\text{water}}}{\mu_{\text{liquid}}}$$

Substituting the given values, we get

$$\frac{(d_{\text{app}})_{\text{liquid}}}{9} = \frac{4/3}{1.5}$$

$$\Rightarrow \quad (d_{\text{app}})_{\text{liquid}} = \frac{4 \times 9 \times 2}{3 \times 3} = 8\text{ cm}$$

23. *(c)* Given, object distance, $u = -10\text{ cm}$

Image distance, $v = -20\text{ cm}$

For concave mirror, magnification produced,

$$m = \frac{-v}{u} = -\left(\frac{-20}{-10}\right) = -2$$

24. *(d)* Given,

$$\frac{\beta_1}{\beta_2} = \frac{3}{2}$$

As, fringe width in the case of YDSE is given as

$$\beta = \frac{\lambda D}{d} \quad \text{[here, } D \text{ and } d \text{ are constants]}$$

where, λ is the wavelength of light beam.

$$\Rightarrow \quad \beta \propto \lambda$$

$$\Rightarrow \quad \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{3}{2}$$

or $$\lambda_1 : \lambda_2 = 3 : 2$$

25. *(b)* As we know, according to Brewster's law, refractive index,

$$\mu = \tan\theta_p \quad \dots \text{(i)}$$

where, θ_p is polarising angle.

Critical angle,

$$\sin\theta_c = \frac{1}{\mu}$$

or $$\mu = \frac{1}{\sin\theta_c} \quad \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{\sin\theta_c} = \tan\theta_p$$

or $$\tan\theta_p \sin\theta_c = 1$$

26. *(c)* Given, $\phi_A = 3\text{ eV}$, $\phi_B = 4\text{ eV}$,

$(\text{KE})_A = (\text{KE})_B = 1\text{ eV}$, $\lambda_A = 500\text{ nm}$

According to Einstein's photoelectric equation, maximum kinetic energy of photoelectrons,

$$\text{KE} = \frac{hc}{\lambda} - \phi$$

For material A,

$$(\text{KE})_A = \frac{hc}{\lambda_A} - \phi_A$$

Similarly, for material B,

$$(\text{KE})_B = \frac{hc}{\lambda_B} - \phi_B$$

As, $(\text{KE})_A = (\text{KE})_B$

$$\Rightarrow \frac{hc}{\lambda_A} - \phi_A = \frac{hc}{\lambda_B} - \phi_B$$

$$\Rightarrow \frac{hc}{\lambda_A} - \frac{hc}{\lambda_B} = \phi_A - \phi_B$$

$$hc\left(\frac{1}{\lambda_A} - \frac{1}{\lambda_B}\right) = (3-4)\text{ eV}$$

$$\Rightarrow \frac{1}{\lambda_A} - \frac{1}{\lambda_B} = \frac{-1\times 1.6\times 10^{-19}}{6.626\times 10^{-34}\times 3\times 10^8}$$

$$\frac{1}{\lambda_B} = \frac{1}{500\times 10^{-9}} + \frac{1.6\times 10^{-19}}{6.626\times 10^{-34}\times 3\times 10^8}$$

$$= 2\times 10^6 + 8.05\times 10^5$$

$$= 2.805\times 10^6$$

or $\lambda_B = 3.56\times 10^{-7}$

$$\simeq 350\text{ nm}$$

27. *(e)* Given, $p_{\alpha-\text{particle}} = \frac{1}{2} p_{\text{proton}}$

$$\Rightarrow \frac{p_{\alpha-\text{particle}}}{p_{\text{proton}}} = \frac{1}{2} \quad \dots\text{(i)}$$

de-Broglie wavelength is given as, $\lambda = h/p$

where, h is Planck's constant and p is momentum.

$$\Rightarrow \lambda \propto \frac{1}{p} \quad (\because p \text{ is constant})$$

$$\Rightarrow \frac{\lambda_{\alpha-\text{particle}}}{\lambda_{\text{proton}}} = \frac{p_{\text{proton}}}{p_{\alpha-\text{particle}}}$$

$$= \frac{2}{1} \quad \text{[from Eq. (i)]}$$

28. *(d)* When a β^--particle is emitted by a nucleus, its atomic number (Z) is increases by one but mass number remains unchanged, i.e.

$$_ZX^A \xrightarrow{\beta^-\text{-particle}} {}_{Z+1}Y^A + \text{anti-neutrino} + e^-$$

Since, Z = number of protons

= number of electrons.

29. *(*)* Given, $n_1 = 10^{18}$ fissions per second

Power, $P_1 = 300$ MW

$= 300\times 10^6$ W

$P_2 = 360\text{MW} = 360\times 10^6$ W

As, power $= \frac{\text{energy}}{\text{time}}$

$$= \frac{\text{number of fission per second } (n) \times \text{energy per fission}}{\text{time}}$$

If energy per fission and time is constant, then

power $\propto n$

$$\Rightarrow \frac{P_1}{P_2} = \frac{n_1}{n_2}$$

$$\Rightarrow n_2 = \frac{n_1 \times P_2}{P_1}$$

$$= \frac{10^{18}\times 360\times 10^6}{300\times 10^6}$$

$$= 1.2\times 10^{18}$$

30. *(d)* Total energy of electron in H-atom,

$$\text{TE} = \text{PE} + \text{KE} \quad \dots\text{(i)}$$

where, PE and KE are the potential and kinetic energies of electron in H-atom.

As, $\text{PE} = -2\text{KE}$

$\Rightarrow \text{TE} = -2\text{KE} + \text{KE}$ [from Eq. (i)]

$= -\text{KE}$

So, $\text{TE}/\text{KE} = -1$

31. *(d)* Given, $\frac{A_1}{A_2} = \frac{3}{2}$

Since, nuclear density (ρ) is independent of the mass number of the nuclei.

So, $\rho_1 : \rho_2 = 1:1$

32. *(e)* In p-type semiconductors,

(i) holes are majority charge carrier and electrons are minority charge carrier.

(ii) impurity added in acceptor type.

(iii) electrons will move from one hole to another hole constituting a flow of current.

(iv) Vacancy of electron is a hole with positive charge.

33. *(b)* Given, $I_E = 6$ mA, $\alpha = 0.95$

As, current gain in CB, $\alpha = \frac{I_C}{I_E}$

where, I_E is emitter current and I_C is collector current.

$\Rightarrow I_C = \alpha I_E$

$= 0.95\times 6$

$= 5.7$ mA

As, $I_E = I_B + I_C$

where, I_B is base current.

$\Rightarrow I_B = I_E - I_C$

$= 6 - 5.7$

$= 0.3\text{mA}$

34. *(a)* LED is a forward biased *p-n* junction diode which emits light when recombination of electrons and holes take place at the junction.

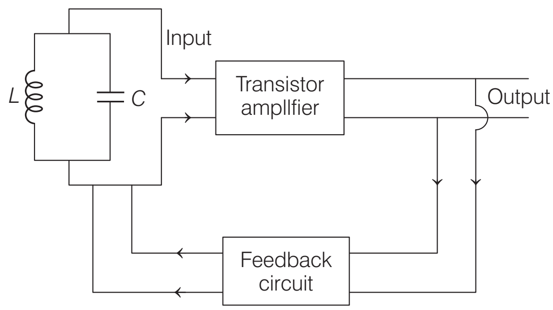
The materials/compounds used for making LED are

(i) Gallium arsenide (GaAs)

(ii) Gallium phosphide

(iii) Gallium arsenide phosphide

35. *(e)* A transistor amplifier along with a tank circuit with positive feedback will act as an oscillator as shown below



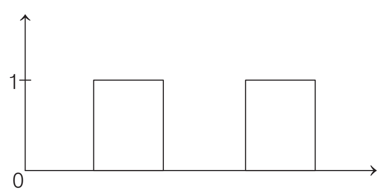
36. *(c)* In a transmitter, we have carrier wave of frequency ω_c and modulated wave of frequency ω_m. Along with these, there are two side bands of frequency $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$.

Thus, the band pass filter will reject the frequency of $2\omega_c$.

37. *(e)* Amongst the given options, statement given in option (e) is incorrect but rest are correct.

It can be corrected as,

digital signals have only two values as shown below



Analog signals are usually in the form of sine waves.

38. *(b)* Given, $A \times v = B$

where, B is energy.

$\Rightarrow A \times v = \text{Energy}$

Dimensional formula of v, $[v] = [LT^{-1}]$

Dimensional formula of E,

$[E] = [ML^2T^{-2}]$

Let dimensional formula of quantity $A = [M^aL^bT^c]$

$\Rightarrow [M^aL^bT^c][LT^{-1}] = [ML^2T^{-2}]$

$\Rightarrow [M^aL^{b+1}T^{c-1}] = [ML^2T^{-2}]$

From the principle of homogeneity,

$a = 1;$

$b + 1 = 2 \Rightarrow b = 1$

$c - 1 = -2 \Rightarrow c = -1$

$\Rightarrow$ Dimensional formula of $A = [ML^0T^{-1}]$

$= [MLT^{-1}]$

This is the dimensional formula of momentum.

39. *(a)* Given, % error in mass $= 1\%$

% error in length $= 2\%$

% error in time $= 3\%$

Acceleration of the particle, $a = \frac{v}{t} = \frac{dx}{(dt)\,t}$

% error in a is,

$$\frac{\Delta a}{a} \times 100 = \frac{\Delta x}{x} \times 100 + 2 \times \frac{\Delta t}{t}$$

$= 2\% + 2 \times 3\%$

$= (2 + 6)\% = 8\%$

40. *(a)* Given, radius, $r = 1.05\,\text{m}$

Area of circular plate,

$A = \pi r^2$

$= 3.14 \times 1.05 \times 1.05$

$= 3.46185\,\text{m}^2$

According to the rule of significant figures for multiplication, the final result should retain the least significant figures as given in the various numerical values.

So, area upto correct significant figures is $3.47\ \text{m}^2$.

41. *(c)* Given, $\mathbf{v} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$

$= v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

where, v_x and v_y are the components of velocity in X and Y-axis, respectively.

So, $v_x = 1$

and $v_y = 1$

Magnitude of $\mathbf{v} = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{1^2 + 1^2}$$
$$= \sqrt{2}$$

Angle made by velocity of the particle with X-axis,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}(1)$$
$$= 45°$$

42. *(b)* Velocity of the hammer dropped into a mine is given as, $v = \sqrt{2gh}$

where, g is acceleration due to gravity and h is depth of mine.

Since, g is constant.

$\Rightarrow \quad v \propto \sqrt{h}$

$\Rightarrow \quad v_d : v_{2d} : v_{3d} = 1 : \sqrt{2} : \sqrt{3}$

43. *(c)* When brakes are applied to a moving vehicle, the distance is travels before stopping is called stopping distance. It is given by

$s = \frac{u^2}{2a}$, where u is initial velocity and a is the retardation produced by brakes.

44. *(a)* If the body starts from rest, i.e. initial velocity $u = 0$.

Since, the body is moving with constant acceleration a.

So,

$$a = \frac{dv}{dt}$$

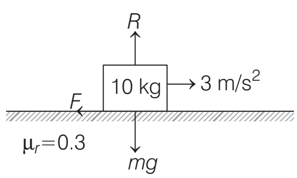
$\Rightarrow \quad \frac{dv}{dt} = \text{constant}$

$\Rightarrow \quad v = 0 \text{ at } t = 0$

So, the v-t graph will be a straight line passing through the origin.

Hence, option (a) is correct.

45. *(d)* FBD of the given block is as shown below



Net force on the block due to which it is moving in horizontal direction,

$$F_{net} = ma$$
$$= 10 \times 3 = 30 \text{ N}$$

Also, $\quad F_{net} = F_{applied} - f$

where, f is frictional force.

$\Rightarrow \quad F_{applied} = F_{net} + f \quad \dots \text{(i)}$

As in equilibrium,

$$\Sigma F_x = 0$$

$\Rightarrow \quad R = mg \quad \dots \text{(ii)}$

Also, $\quad F = \mu_r R = \mu_r mg \quad$ [from Eq. (ii)]

Now, Eq. (i) can be written as

$$F_{applied} = F_{net} + \mu_r mg$$

Substituting the values in the above equation, we get

$$F_{applied} = 30 + 0.3 \times 10 \times 10$$
$$= 30 + 30$$
$$= 60 \text{ N}$$

46. *(a)* All the statements in the options are correct, but it is incorrect in option (a). It can be corrected as,
Acceleration of a particle is zero, if it is at rest or change in velocity in the given interval of time is zero.

However, in uniform linear motion, velocity remains constant.

47. *(d)* Force required to maintain the speed of the belt

$$= \frac{dp}{dt} = \frac{d(mu)}{dt} = u\left(\frac{dm}{dt}\right)$$

where, u is the speed with which belt is moving and $\frac{dm}{dt}$ is the rate which sand is falling.

48. *(c)* Area under the force-time graph, i.e. $F \times \Delta t$.

This quantity gives the change in momentum or impulse.

So, $\underbrace{Fdt}_{\text{Impulse}} = \underbrace{p_2 - p_1}_{\text{Change in momentum}}$

= Curve under F-Δt graph.

49. *(b)* When a metal spring is stretched, work is done against the interatomic force. This work is stored in metal spring in the form of elastic potential energy.

50. *(b)* Instantaneous power, $P = \lim_{\Delta t \to 0} \frac{\text{Work done}}{\text{Time taken}}$

$$= \left|\frac{dW}{dt}\right|$$

As, $dW = \mathbf{F} \cdot d\mathbf{r}$

$\Rightarrow \quad P = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$

Again, $\frac{d\mathbf{r}}{dt} = \mathbf{v}$, instantaneous velocity

Therefore, $P = \mathbf{F} \cdot \mathbf{v} = Fv\cos\theta$

51. *(c)* According to the question,
kinetic energy of the ball = energy in the spring

$\Rightarrow \quad (\text{KE})_{\text{ball}} = \frac{1}{2}kx^2$

where, $(\text{KE})_{\text{ball}} = 10^3$ J (given)

x is the compression in the spring = 50 cm
= 0.5 m (given)

and k is spring constant.

$\Rightarrow \quad 10^3 = \frac{1}{2} \times k \times (0.5)^2$

or $\quad k = \frac{2 \times 10^3}{0.25}$

or $\quad k = 8 \times 10^3 \text{ Nm}^{-1}$

52. *(b)* Given, $h_1 = h$

and $\quad h_2 = \frac{h}{4}$

Let m is the mass of the object.
Initial potential energy, $U_i = mgh$
Just before it strikes the ground, its initial potential energy is converted into kinetic energy.

$\Rightarrow \quad \text{KE} = U_i$

When it rebounds to height h_2, so final

$$\text{PE} = mgh_2 = \frac{mgh}{4} \quad \left(\because h_2 = \frac{h}{4}\right)$$

So, the fraction of energy loss $= \frac{mgh - \frac{mgh}{4}}{mgh} = \frac{3}{4}$

53. *(c)* Rotational kinetic energy is given as

$$K_R = \frac{1}{2}I\omega^2$$

where, I is moment of inertia and ω is angular frequency.

As, mass and radius of ring and disc is same.

So, $\quad I_{\text{ring}} = \frac{1}{2}MR^2$

and $\quad I_{\text{disc}} = \frac{1}{4}MR^2$

$\therefore$ The required ratio is,

$$\frac{(K_R)_{\text{ring}}}{(K_R)_{\text{disc}}} = \frac{\frac{1}{2}MR^2}{\frac{1}{4}MR^2} = \frac{2}{1}$$

54. *(d)* Given, $m_1 = m$, $m_2 = 2m$, $m_3 = 3m$

x-coordinates of these particles are,

$x_1 = 0$, $x_2 = 1$, $x_3 = -2$

$\therefore$ x-coordinates of centre of mass of the system are,

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + 2m \times 1 + 3m \times (-2)}{m + 2m + 3m}$$

$$= \frac{2m - 6m}{6m} = \frac{-4m}{6m}$$

$$= -\frac{2}{3}$$

55. *(b)* If a body has mass M and radius of gyration is K, then moment of inertia is, $I = MK^2$

or $\quad K = \sqrt{\frac{I}{M}}$

For a solid cylinder about its long axis of symmetry,

$$I = \frac{1}{2}MR^2$$

Putting this value in above equation, we get

$\Rightarrow \quad K = \sqrt{\frac{\frac{1}{2}MR^2}{M}}$

$\Rightarrow \quad K = \frac{R}{\sqrt{2}}$

56. *(d)* According to the law of conservation of angular momentum, when no external torque acts on rotating object, angular momentum remains conserved.

However, rotational kinetic energy is given as

$$(\text{KE})_R = \frac{1}{2}I\omega^2$$

where, I is moment of inertia and ω is angular frequency.

$\Rightarrow \quad (\text{KE})_R \propto I$

57. *(c)* According to Kepler's third law, the square of time period (T) of revolution of a planet around the sun is directly proportional to the cube of semi-major axis (d) of its elliptical path, i.e. $T^2 \propto d^3$.

58. *(c)* Acceleration due to gravity, $g = \frac{GM}{R^2}$

where, R is radius of the earth, M is the mass of the earth and G is gravitational constant.

$$\Rightarrow \quad g \propto \frac{M}{R^2}$$

...(i) ($\because G$ is constant)

Let new acceleration due to gravity be g, new radius of earth is $R' = R/2$ and new mass of earth is $M' = \frac{M}{2}$

Now, $$g' \propto \frac{M'}{R'^2} \quad ...(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\Rightarrow \quad \frac{g'}{g} = \frac{M'}{M}\frac{R^2}{R'^2}$$

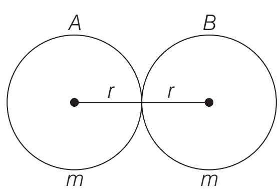
Given, $M' = \frac{M}{2}$

and $R' = \frac{R}{2}$

$$\Rightarrow \quad \frac{g'}{g} = \frac{M/2}{M}\frac{(R)^2}{(R/2)^2}$$

or $g' = 2g$

59. *(d)* According to the universal law of gravitation, force between two identical spheres each of radius r, is given as



$$F = \frac{km \times m}{(2r)^2}$$

$$= \frac{km^2}{4r^2} \Rightarrow F \propto \frac{1}{r^2}$$

60. *(e)* Viscosity is the internal friction in a fluid which occurs due to diffusion of molecules of one layer into other. Viscous force opposes the motion of one portion of a fluid relative to other.

So, viscous force is due to cohesion between the fluid molecules and transfer of momentum between them. Surface tension is the property of the liquid at rest by virtue of which a liquid tends to occupy minimum surface area and behaves like stretched membrane. It also depends upon the cohesive force.

Since, intermolecular cohesive force decreases with the rise in temperature.

Hence, with the increase in temperature, viscosity and surface tension of the liquid decreases.

61. *(e)* Statement given in option (e) is true but rest are false.

The reciprocal of the bulk modulus (B) of a body is called the compressibility (C) of that material, i.e.

$$\boxed{C = \frac{1}{B}}$$

However, Young's modulus (Y) of a wire is independent of its length.
Unit of Y is Nm^{-2}. Dimensional formula of stress is same as that of pressure.

Strain is a unitless and it is dimensionless physical quantity.

62. *(c)* When a body is strained, work is done against the interatomic forces. This work is stored in the wire in the form of elastic potential energy.

So, stored elastic energy,

$$U = \frac{1}{2} Y (\text{strain})^2 \times \text{volume}$$

where, Y is Young's modulus.

$\therefore$ Energy stored per unit volume

$$= \frac{1}{2} Y (\text{strain})^2 \quad ...(i)$$

We know that, Young's modulus,

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

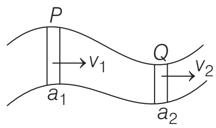
$$\Rightarrow \quad \text{Strain} = \frac{\text{Stress}}{Y} \quad ...(ii)$$

Putting Eq. (ii) in Eq (i), we get
Energy stored per unit volume

$$= \frac{1}{2}\frac{(\text{Stress})^2}{Y}$$

63. *(c)* According to equation of continuity, when an incompressible and non-viscous fluid flows in a streamlined motion through a tube of non-uniform cross-section, then mass of rate of flow is same at every section of tube.

Thus,



$$a_1v_1 = a_2v_2$$

or $\quad av =$ constant.

64. *(d)* When the stopcock is suddenly opened, then there will be no change in the internal energy of the gas, as no work is done by the gas.

Since, no work is being done by the gas, so temperature of the gas will remain constant.

65. *(a)* An **adiabatic process** is a thermodynamic process in which there is no flow of heat between the system and the surroundings.

However, a process taking place in a thermodynamic system at constant temperature is called **isothermal process.**

A process taking place in a thermodynamic system at constant pressure is called **isobaric process.**

A process taking place in a thermodynamic system at constant volume is called **isochoric process.**

When a thermodynamic system returns to its initial state after passing through several states, then it is called a **cyclic process.**

66. *(b)* Given, temperature of source, $T_1 = 400$ K

Initial efficiency, $\eta = 25\%$

Final efficiency, $\eta' = 50\%$

As, efficiency of the Carnot cycle,

$$\eta = 1 - \frac{T_2}{T_1}$$

where, T_2 is the temperature of the sink.

Let $\quad T_2 = T$

$\therefore$ Initial efficiency, $\eta = 1 - \frac{T}{T_1} = 25\%$

Substituting the given values, we get

$$\frac{25}{100} = 1 - \frac{T}{400}$$

$$4 \times 25 = 400 - T$$

$\Rightarrow \quad T = 400 - 100 = 300$ K

Final efficiency, $\eta' = 1 - \frac{T}{T_1'} = 50\%$

$\Rightarrow \quad 1 - \frac{300}{T_1'} = \frac{50}{100}$

$\Rightarrow \quad 100(T_1' - 300) = T_1' \times 50$

or $\quad 100\, T_1' - 30000 = 50 T_1'$

or $\quad 50 T_1' = 30000$

$\Rightarrow \quad T_1' = 600$ K

67. *(a)* The desired fraction,

$$F = \frac{\Delta U}{\Delta Q} = \frac{nC_V \Delta T}{nC_p \Delta T}$$

$$= \frac{C_V}{C_p} = \frac{1}{\gamma}$$

As, for diatomic gas, $\gamma = \frac{7}{5}$

$\Rightarrow \quad F = \frac{5}{7}$

68. *(*)* Average kinetic energy of n moles of a gas

$$= \frac{n}{2} fRT$$

where, f is the degree of freedom.

Since, H_2 and N_2 both are diatomic in nature, so their degree of freedom will be same.

Number of moles of $H_2, n_1 = \frac{m_1}{m_{H_2}}$

where, m_1 is given mass = 4 g

and m_{H_2} is molar mass = 2 g

$\Rightarrow \quad n_1 = \frac{4}{2} = \frac{2}{1}$

Similarly, number of moles of N_2,

$$n_2 = \frac{m_2}{m_{N_2}}$$

where, m_2 is given mass = 7 g

and m_{N_2} is molar mass = 28 g

$\Rightarrow \quad n_2 = \frac{T}{28} = \frac{1}{4}$

$\therefore$ The ratio of kinetic energy of H_2 and N_2

$$= \frac{\frac{n_1}{2} fRT}{\frac{n_2}{2} fRT}$$

$$= \frac{\frac{2}{1}}{\frac{1}{4}} = \frac{2}{1} \times \frac{4}{1} = 8:1$$

69. *(a)* Minimum speed of air molecules, so that they could escape the planet is given as

$$v_e = \sqrt{2gR}$$

$$= 1.414\sqrt{gR}$$

So, the planet will have atmosphere only, if rms speed of molecules of air is $1.414\sqrt{gR}$.

70. *(d)* Given,

$$\frac{\text{acceleration due to gravity on earth, } g_e}{\text{acceleration due to gravity on moon, } g_m} = \frac{6}{1} \quad \dots \text{(i)}$$

Time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T \propto \sqrt{\frac{1}{g}}$$

$$\Rightarrow \quad \frac{T_e}{T_m} = \sqrt{\frac{g_m}{g_e}}$$

where, T_e and T_m is the time period on the surface of earth and moon, respectively.

Using Eq. (i), we get

$$\frac{T_e}{T_m} = \sqrt{\frac{1}{6}} = \frac{1}{\sqrt{6}}$$

71. *(e)* Given, $y = 0.5 \sin\left(\frac{\pi}{2}t + \frac{\pi}{3}x\right)$

Comparing the above equation with standard equation of transverse wave, which is $y = A\sin(\omega t + kx)$, we get

Angular frequency, $\omega = \frac{\pi}{2}$

Propagation constant, $k = \frac{\pi}{3}$

As, wave number, $K = \frac{2\pi}{\lambda} = \frac{\omega}{v}$

where, λ is the wavelength of the wave and v is the wave velocity.

Substituting the given values in Eq. (i), we get

$$\frac{\pi}{3} = \frac{\pi/2}{v}$$

$$\Rightarrow \quad v = \frac{3}{2} = 1.5 \text{ ms}^{-1}$$

72. *(a)* Kinetic energy of the particle executing simple harmonic motion is given as

$$\text{KE} = \frac{1}{2}m\omega^2(a^2 - x^2)$$

where, a is amplitude and x is displacement.

Given, $\text{KE} = \frac{1}{4}ma^2\omega^2$

$$\Rightarrow \quad \frac{1}{4}m\omega^2a^2 = \frac{1}{2}m\omega^2(a^2 - x^2)$$

$$\text{or} \quad \frac{1}{2}a^2 = a^2 - x^2$$

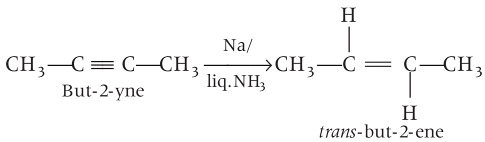
$$\Rightarrow \quad x^2 = a^2 - \frac{1}{2}a^2$$

$$x^2 = \frac{1}{2}a^2$$

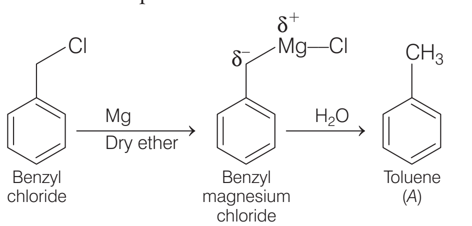
$$\text{or} \quad x = \frac{a}{\sqrt{2}}$$

Chemistry

73. *(d)* When but-2-yne is treated with Na/liquid NH_3, addition of two H-atoms takes place at the triple bonded carbon atoms in *trans*-form is



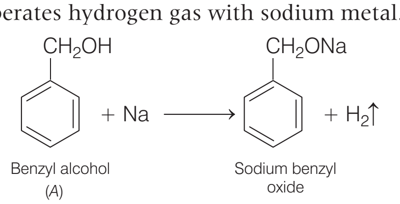
74. *(a)* When benzyl chloride is reacted with magnesium metal in the presence of dry ether, then benzyl magnesium chloride is formed. On further treating with water, toluene is formed which is compound '*A*'.



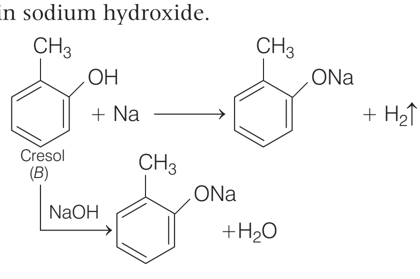
75. *(c)* The boiling point rises with increase in molecular mass and size of halogen atom as the magnitude of van der Waals' forces increases. So, the correct increasing order of boiling point is

$$CH_3Cl < CH_3Br < CH_2Br_2 < CHBr_3$$

76. *(b)* Compound *A* is benzyl alcohol because it liberates hydrogen gas with sodium metal.

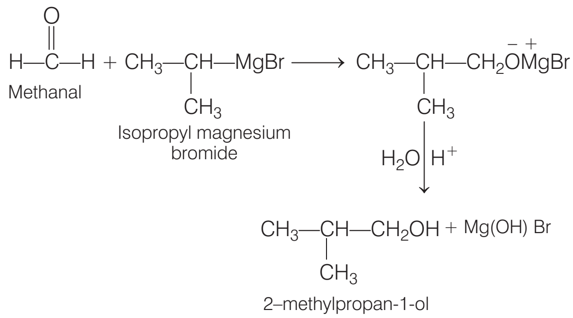


Also, benzyl alcohol do not dissolve in sodium hydroxide. Compound *B* is cresol as it liberates hydrogen gas with sodium and further dissolves in sodium hydroxide.

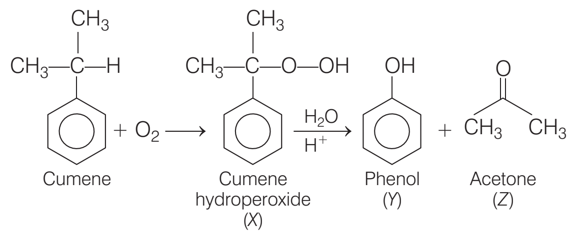


Compound *C* is anisole (C_7H_8O) as it does not react with Na metal and NaOH.

77. *(e)* Isopropyl magnesium bromide is used for preparation of 2-methyl propan-1-ol from methanal as follows.



78. *(d)* When cumene is oxidised in the presence of air, it gives cumene hydroperoxide (*X*) and on further hydrolysis, it gives phenol (*Y*) and acetone (*Z*).



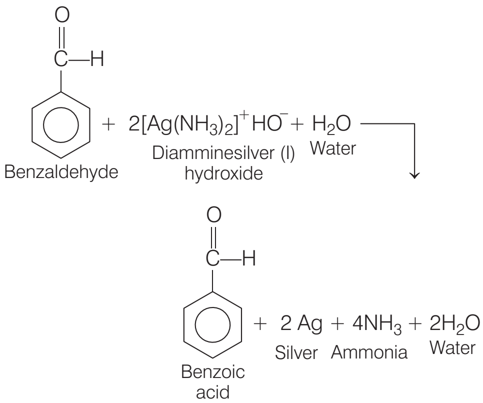
79. *(b)* Iodoform test is given by those compounds which has methyl ketone moiety. Hence, *P* is acetone.

$$\underset{\text{Acetone }(P)}{CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3} \xrightarrow{I_2/NaOH} \underset{\text{Sodium acetate}}{CH_3COONa} + \underset{\text{Iodoform}}{CHI_3}$$

Fehling's solution gives red precipitate with aliphatic aldehydes. So, *R* is acetaldehyde.

$$\underset{\text{Acetaldehyde}}{CH_3-CHO} + \underbrace{2Cu^{2+} + OH^-}_{\text{Fehling's solution}} \longrightarrow \underset{\text{Acetic acid}}{CH_3COOH} + \underset{\text{Cuprousoxide (Red ppt.)}}{Cu_2O}$$

Tollen's test is given by aldehydes, aromatic aldehydes and α-hydroxy ketones. So, *Q* is benzaldehyde.



The remaining compound S did not give any test. So, it is diethyl ketone.

Therefore, P = Acetone, Q = Benzaldehyde, R = Acetaldehyde and S = Diethyl ketone

80. *(a)* The acidity of carboxylic acid depends on the electron withdrawing groups attached to carbon atoms. At the same time, the distance between the electron withdrawing groups from carboxylic acid is also responsible for the acidic strength of carboxylic acids. As the distance of electron withdrawing group from carboxylic acid increases, the strength of carboxylic acid decreases. So,

$Cl—CH_2—CH_2—COOH < Cl—CH_2—COOH$

The electron withdrawing groups increases the acidic strength of carboxylic acids.

$\therefore Cl—CH_2—CH_2—COOH < Cl—CH_2—COOH <$
$NC—CH_2—COOH < Cl—CH(Cl)—COOH$

81. *(c)* Instead of ethanamide, propanamide reacts with Br_2 in the presence of sodium hydroxide to give ethylamine.

$$\underset{\text{Propanamide}}{CH_3CH_2CONH_2} \xrightarrow[\text{NaOH}]{Br_2} \underset{\text{Ethanamine or ethylamine}}{CH_3CH_2NH_2} + CO_2$$

This reaction is known as Hofmann degeadation reaction.

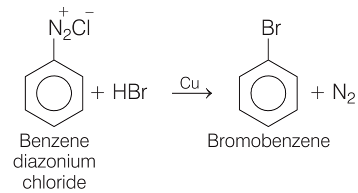
So, (c) is incorrect about amines.

82. *(a)* The ammonium compounds exhibit the following order of solubility in water.

Primary ammonium (RNH_3^+) > secondary ammonium ($R_2NH_2^+$) > tertiary ammonium (R_3NH^+)

Quaternary ammonium salts exhibit the lowest extent of H-bonding of alkyl ammonium ion in water. The reason behind this is more the number of hydrogen bonding, more is the hydration energy of molecule and hence, more is the stability of the ammonium compound.

83. *(b)* The reaction involving the treatment of benzene diazonium chloride with HBr in the presence of copper powder is known as Gattermann's reaction



84. *(c)* Pentaacetate of glucose does not react with NH_2OH because it does not form an open chain structure and indicates the absence of free—CHO group.

85. *(d)* Fibrous protein present in muscles is myosin. The fibrous proteins have polypeptide chains arranged in long strands which are present in the majority of the bones and blood vessels.

86. *(a)* Anti-depressant drugs inhibit enzymes that catalyse noradrenaline degradation. These drugs are phenelzine and iproniazid which slow down the metabolism of noradrenaline.

87. *(b)* Greenhouse gases like CO_2, methane, etc., are responsible for global warming. Because greenhouse gases are capable of absorbing infrared radiation, thereby trapping heat in atmosphere and increase the heat in the atmosphere.

88. *(a)* Volume of 11.2 L of gas $\longrightarrow$ 22 g at STP

Volume of 22.4 L of gas $\longrightarrow$ 44 g

$\therefore$ The mass of C_3H_8 is 44 g. Hence, cooking gas contains propane (C_3H_8).

89. *(b)* Maximum number of electrons in 3rd energy level $= 2 \times n^2 = 2 \times (3)^2 = 18$ electrons.

Half of them have $m_s = +1/2$, i.e. $\frac{1}{2} \times 18 = 9$ electrons.

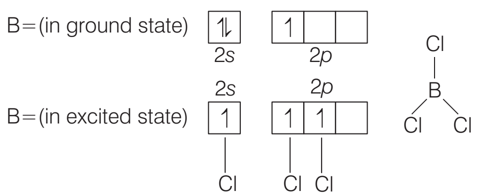
90. *(b)* Pauli's exclusion principle states that, ''in an atom or molecule, no two electrons can have the same set of four quantum numbers''.

91. *(e)* As ionisation enthalpy decreases from top to bottom in a group and increases from left to right in a period, hence, bismuth has the least value of first ionisation enthalpy.

92. *(a)* When a liquid crystallises into a solid, then water molecules are constrained and an ordered state is obtained and hence, entropy decreases.

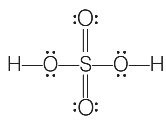
93. *(b)* BCl_3

$B = 1s^2\ 2s^2\ 2p^1$



$\therefore$ BCl_3 involves sp^2 hybridisation and trigonal plannar geometry.

94. *(e)* In sulphuric acid, the central atom, S has expanded octet. It has 12 valence electrons.



95. *(c)* According to Boyle's law, $p_1V_1 = p_2V_2$

$$p_1 = 1 \text{ bar}$$
$$p_2 = ?$$
$$V_1 = 500 \text{ mL}$$
$$V_2 = 1\text{L} = 1000 \text{mL}$$

$\therefore 1 \text{ bar} \times 500 \text{ mL} = p_2 \times 1000 \text{ mL}$

$$p_2 = \frac{1 \text{ bar} \times 500\text{mL}}{1000 \text{ mL}} = 0.5 \text{ bar}.$$

96. *(e)* The ratio of molar volume to ideal molar volume is called compressibility factor, Z.

$$Z = \frac{V_m}{V_{\text{ideal}}}$$

97. *(e)* Combustion reaction will always occur when a substance reacts with oxygen from air to release energy. Since, heat is released for combustion reaction, the enthalpy change of the system, ΔH will be negative always.

98. *(c)* $\Delta H_{\text{vap}} = \Delta U + \Delta n_g RT$

where, ΔH_{vap} = enthalpy change of vaporisation

ΔU = internal energy

$\Delta n_g = n_2 - n_1$ = difference between number of moles of reactant and product.

$$H_2O(l) \longrightarrow H_2O(g)$$

$\therefore \quad \Delta n_g = 1 - 0 = 1$

$T = 127°C = 127 + 273 = 400 \text{ K}$

$R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$

$\therefore \Delta H_{\text{vap}} = \Delta U + \Delta n_g RT$

$\Rightarrow \Delta U = \Delta H_{\text{vap}} - \Delta n_g RT$

$= 40 \times 10^3 - 1(8.3)(400) = 36680 \text{ Jmol}^{-1}$

$= 36.68 \text{ kJmol}^{-1}$

$\approx 37 \text{ kJ mol}^{-1}$.

99. *(d)* (a) $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$

$\Delta n_g = 2 + 1 - (2) = 1$

(b) $Ni(s) + 4CO(g) \rightleftharpoons Ni(CO)_4(g)$

$\Delta n_g = 1 - (0 + 4) = 1 - 4 = -3$

(c) $CO_2(g) + C(s) \rightleftharpoons 2CO(g)$

$\Delta n_g = 2 - (1 + 0) = 1$

(d) $H_2(g) + Br_2(g) \rightleftharpoons 2HBr(g)$

$\Delta n_g = 2 - (1 + 1) = 2 - 2 = 0$

(e) $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$\Delta n_g = 2 - 1 = 1$

Hence, reaction (d) has zero Δn_g value.

100. *(a)* $[N_2] = 1 \times 10^{-2}$ M

$[H_2] = 2 \times 1^{-2}$ M

$[NH_3] = 2 \times 10^{-2}$ M

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

Equilibrium constant, $K_c = \frac{[NH_3]^2}{[N_2][H_2]^3}$

$$\therefore \quad K_c = \frac{(2 \times 10^{-2})^2}{(1 \times 10^{-2})(2 \times 10^{-2})^3}$$

$$= \frac{(2 \times 10^{-2})^{2-3}}{1 \times 10^{-2}} = \frac{1}{(1 \times 10^{-2})(2 \times 10^{-2})}$$

$$= 0.5 \times 10^4 \text{ mol}^{-2} \text{ dm}^6$$

$$= 5 \times 10^3 \text{ mol}^{-2} \text{ dm}^6$$

101. *(d)* Molar conductivity is expressed as

$$\Lambda_m = \frac{\kappa}{C}$$

[where, κ = specific conductivity and C = concentration]

Here, the unit of κ is S/m and unit of C is mol/L.

So, SI unit of molar conductivity is S m^2 mol^{-1}.

102. *(e)* Disproportionation redox reaction involves simultaneous oxidation and reduction of atoms of same element from one oxidation state to two different oxidation states.

$$2\overset{+4}{N}O_2(g) + 2OH^- \longrightarrow \overset{+3}{N}O_2^- + \overset{+5}{N}O_3^- + H_2O$$

In this reaction, N of nitrogen dioxide is of +4 oxidation state but after the reaction, N is present in +3 and +5 oxidation state simultaneously. So, this reaction is an example of disproportionation redox reaction.

103. *(d)* $\Delta T_b = K_b m$

or $\Delta T_b = \dfrac{K_b \times w \times 1000}{M \times W}$

Where $\Delta T_b = T_b^\circ - T_b$

$= 100 - 98.98 = 1.02$

K_b = molal elevation or ebullioscopic constant

w = weight of the solute

M = molar mass of solute

W = weight of solvent (in grams)

Now, $\Delta T_b = \dfrac{0.51 \times w \times 1000}{60 \times 2}$

$$1.02 = \frac{0.51 \times w \times 1000}{120}$$

$$w = \frac{1.02 \times 120}{0.51 \times 1000}$$

$$= \frac{122.4}{510} = 0.24 \text{ or } w = 240\text{ g}$$

So, 240 g of urea is to be added to 2 kg of water to get the boiling point 100°C at same place.

104. *(a)* we know, $\dfrac{p^\circ - p}{p^\circ} = \chi_B = \dfrac{n_B}{n_A} = \dfrac{\dfrac{w_B}{M_B}}{\dfrac{w_A}{M_A}}$

where, p° = vapour pressure of pure benzene = 0.850 bar

p = vapour pressure of solution = 0.845 bar

χ_B = mole fraction of solute

n_B = number of moles of solute

n_A = number of moles of benzene

w_B = mass of solute = 1 g

w_A = mass of benzene = 39 g

M_A = molar mass of benzene = 78 g/mol

M_B = molar mass of solute = ?

$$\therefore \quad \frac{p^\circ - p}{p^\circ} = \frac{w_B \times M_A}{M_B \times w_A}$$

$$= \frac{0.850 - 0.845}{0.850} = \frac{1 \times 78}{M_B \times 39}$$

$$\Rightarrow \quad M_B = \frac{78 \times 0.850}{0.005 \times 39} = 340\text{ g mol}^{-1}$$

105. *(c)* Rate of reaction $= \dfrac{d[P_2Q]}{dt} = -\dfrac{1}{2}\dfrac{d[P]}{dt} = \dfrac{-d[Q]}{dt}$

Now, rate of disappearance of $P = \dfrac{d[P]}{dt}$

We know, $\dfrac{d[P_2Q]}{dt} = -\dfrac{1}{2}\dfrac{d[P]}{dt}$

$$-2\frac{d[P_2Q]}{dt} = \frac{d[P]}{dt}$$

$$\frac{d[P]}{dt} = -2(0.24)$$

$$\left[\text{Given, } \frac{d[P_2Q]}{dt} = 0.24 \text{ mol dm}^{-3}\text{ s}^{-1}\right]$$

Rate of disappearance of $P = -0.48$ mol dm^{-3} s^{-1}

Similarly,

Rate of disappearance of $Q = \dfrac{d[Q]}{dt}$

We know, $\dfrac{d[P_2Q]}{dt} = -\dfrac{d[Q]}{dt}$

$$\frac{d[Q]}{dt} = \frac{-d(P_2Q)}{dt} = -0.24$$

Rate of disappearance of $Q = -0.24$ mol dm^{-3} s^{-1}.

106. *(e)* Statements (ii), (iii) and (v) are correct whereas statements (i) and (iv) are incorrect.

(i) Thermal decomposition of HI on gold surface is zero order reaction.

(ii) Thermal decomposition of $N_2O_5(g)$ at constant volume is first order reaction.

(iii) Hydrogenation of ethene is first order reaction.

(iv) Decomposition of NH_3 on a hot platinum surface is a zero order reaction.

(v) Thermal decomposition of $SO_2Cl_2(g)$ at constant volume is a first order reaction.

107. *(c)* Among the given options, only option (c) is true as both physisorption and chemisorption depend on the nature of the gas (adsorbate). The correction of incorrect options is as follows

(a) Chemisorption is highly specific in nature.

(b) Physisorption is reversible.

(d) Chemisorption has a higher enthalpy of adsorption than physisorption.

(e) Physisorption is a physical adsorption process which also depends on the surface area of the adsorbent.

108. *(d)* Zinc metal reacts with sodium hydroxide and water, then sodium zincate and hydrogen gas are formed.

$$Zn(s) + 2NaOH(aq) + 2H_2O(l) \longrightarrow \underset{\text{Sodium zincate}}{Na_2Zn(OH)_4(aq)} + H_2(g)$$

109. *(b)* '*syn*-gas' or synthesis gas is fuel gas which consist of mixture of hydrogen and carbon monoxide. It is derived from gasification of a carbon containing fuel to a gaseous product.

110. *(e)* Statement (i) and (ii) are correct and other statements are incorrect. The correct statements are as follows

(iii) PCl_5 has trigonal bipyramidal structure.

(iv) All five bonds are not equivalent. It has two axial P—Cl bonds and three equitorial P—Cl bonds, which are of different bond lengths.

111. *(e)* (A) Silicones are used as water proofing of fabrics.

(B) Zeolites are used for cracking of hydrocarbons.

(C) Quartz is used as piezoelectric material.

(D) Borax is used as flux for soldering metals.

(E) Boron fibres are used as light composite material for aircraft.

So, correct match is (e).

112. *(d)* Orthoboric acid is sparingly soluble in cold water but freely soluble in hot water.

113. *(a)* The metal with atomic number 24 have electronic configuration, $3d^5$, $4s^1$.

$M(Z = 24) = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^5\ 4s^1$

$M^{3+} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^3$

$\therefore$ Number of unpaired electrons, $n = 3$

$\therefore$ Magnetic moment, $\mu = \sqrt{n(n+2)} = \sqrt{3(3+2)}$

$= \sqrt{15} = 3.87$ BM

114. *(b)* Scandium exhibits only +3 oxidation state as it can form Sc^{3+} ion to achieve a noble gas configuration ([Ne] $3s^2\ 3p^6$).

115. *(c)* Cr (chromium) contains more number of unpaired electrons. As a result, metallic bond formed will be stronger. Hence, more will be its melting point.

116. *(c)*

	Complex	Conductivity of electrolyte in aqueous solution
(a)	$[Co(NH_3)_6]Cl_3$	1 : 3
(b)	$[Co(NH_3)_4Cl_2]Cl$	1 : 1
(c)	$[Co(NH_3)_5Cl]Cl_2$	1 : 2
(d)	$[Cr(H_2O)_3(NH_3)_3]Cl_3$	1 : 3
(e)	$[Ag(NH_3)_2]^+[Ag(CN)_2]$	1 : 1

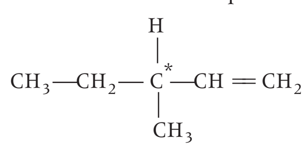
So, pentaammine chlorocobalt (III) chloride $\{[Co(NH_3)_5Cl]Cl_2\}$ corresponds to 1 : 2 electrolyte in aqueous solution.

117. *(b)* The film developed in black and white photography is washed to remove exhausted chemicals from the emulsion. Hence, $[Ag(S_2O_3)_2]^{3-}$ ion is formed after washing.

118. *(a)* Kaolinite is an ore of aluminium. It's chemical composition is $Al_2Si_2O_5(OH)_4$.

119. *(d)* Kjeldahl's method is not applicable to those nitrogen compounds which contains nitro and diazo groups in rings such as pyridine, quinoline, etc., because these compounds do not convert to ammonium sulphate under the conditions of this method.

120. *(c)* 3-methyl-1-pentene exhibits optical isomerism because it possesses a chiral carbon.



(C^* = Chiral carbon)

Mathematics

1. *(c)* Given, $19 \le -3x \le 27 \;\forall\; x \in Z$

$$-9 \le x \le -\frac{19}{3}$$

$$-9 \le x \le -6.33$$

$$x = \{-9, -8, -7\}$$

2. *(d)* Given, $X = \{1, \pi, \{42, \sqrt{2}\}, \{1, 3\}\}$

P : $\pi \in X$. This statement is true.

Q : $\{1, 3\} \subseteq X$. This statement is false as $\{1, 3\} \in X$.

R : $\{1, \pi\} \subseteq X$. This statement is true.

3. *(a)* Given, $0 \le \theta \le \frac{\pi}{2}$ and $\sin\left(\theta + \frac{\pi}{6}\right) = \cos\theta$

$$\sin\left(\theta + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2} - \theta\right)$$

Comparing the angles on both sides,

$$\theta + \frac{\pi}{6} = \frac{\pi}{2} - \theta$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{2} - \frac{\pi}{6}$$

$$\Rightarrow \quad 2\theta = \frac{2\pi}{6}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6}$$

4. *(b)* $\operatorname{cosec}\theta + \cot\theta = 5 \quad \ldots \text{(i)}$

As, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$$

$$\Rightarrow \quad \operatorname{cosec}\theta - \cot\theta = \frac{1}{5} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii),

$$2\cot\theta = 5 - \frac{1}{5}$$

$$\cot\theta = \frac{24}{5 \times 2}$$

$$\therefore \quad \tan\theta = \frac{5}{12}$$

5. *(c)* To determine the value of $\tan^{-1}\left(\frac{7}{4}\right) - \tan^{-1}\left(\frac{3}{11}\right)$, use the formula

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right)$$

$$\therefore \tan^{-1}\left(\frac{7}{4}\right) - \tan^{-1}\left(\frac{3}{11}\right) = \tan^{-1}\left(\frac{\frac{7}{4} - \frac{3}{11}}{1 + \frac{7}{4} \times \frac{3}{11}}\right)$$

$$= \tan^{-1}\left[\frac{77 - 12}{44 + 21}\right]$$

$$= \tan^{-1}\left[\frac{65}{65}\right]$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

6. *(d)* Given, $0 < \theta < \frac{\pi}{2}$ and $\tan\theta = \frac{\sqrt{5}}{2}$

$$\sec\theta = \sqrt{1 + \tan^2\theta}$$

$$= \sqrt{1 + \frac{5}{4}} = \frac{3}{2}$$

$$\therefore \quad \cos\theta = \frac{2}{3} \quad \left[\because \cos\theta = \frac{1}{\sec\theta}\right]$$

7. *(b)* $\sin^2\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$

As, $\cos^{-1}\frac{3}{5}$ can be written as $\sin^{-1}\frac{4}{5}$.

$$\therefore \quad \sin^2\left(\sin^{-1}\frac{4}{5}\right) = \left(\frac{4}{5}\right)^2$$

$$= \frac{16}{25}$$

8. *(b)* $\cos^4\frac{\pi}{12} - \sin^4\frac{\pi}{12}$

$$= \left(\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}\right)\left(\cos^2\frac{\pi}{12} + \sin^2\frac{\pi}{12}\right)$$

$$= \left(\cos\frac{2\pi}{12}\right)(1)$$

$$= \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

9. *(c)* $\tan\left(2\tan^{-1}\left(\frac{2}{5}\right)\right)$

Using the formula,

$$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$$

$$\tan\left(2\tan^{-1}\left(\frac{2}{5}\right)\right) = \tan\left(\tan^{-1}\left[\frac{2 \times \frac{2}{5}}{1 - \left(\frac{2}{5}\right)^2}\right]\right)$$

$$= \frac{\frac{4}{5}}{\frac{21}{25}} = \frac{20}{21}$$

10. *(a)* $\sin 2x = \frac{\sqrt{3}}{2}\ \forall\ x \in [0, \pi]$

If $x \in [0, \pi]$, $2x \in [0, 2\pi]$

$\sin 2x = \sin\frac{\pi}{3}$ or $\sin 2x = \sin\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow \quad 2x = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = \frac{\pi}{6}, \frac{\pi}{3}$

11. *(e)* Given, $\sin\alpha + \sin\beta = \frac{\sqrt{6}}{2}$ ... (i)

$\cos\alpha + \cos\beta = \frac{\sqrt{2}}{2}$ ... (ii)

Squaring and adding both equations,

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$

$+ \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = \frac{6}{4} + \frac{2}{4}$

$1 + 1 + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 2$

$\cos(\alpha - \beta) = 0$

12. *(c)* Equation of line passing through the points $(-5, -2)$ and $(4, 7)$ is

$y + 2 = \left(\frac{7+2}{4+5}\right)(x + 5)$

$y + 2 = x + 5$

$y = x + 3$

Comparing this equation with the given equation $ay = x + b$ to determine the value of a and b.

Thus, $a = 1$, $b = 3$

Hence, $2a + b = 2 \times 1 + 3 = 5$

13. *(d)* Let equation of line

$y = mx + c$ ... (i)

Since, Eq. (i) is passing through $(2, 5)$ having slope $\frac{1}{2}$, then

$\therefore \quad 5 = \frac{1}{2}(2) + c$

$\Rightarrow \quad c = 5 - 1 = 4$

14. *(a)* Slope of line segment joining the points $(10, 0)$ and $(0, -4)$

$= \frac{-4-0}{0-10} = \frac{4}{10} = \frac{2}{5}$

Mid point of line segment joining the points $\left(\frac{10+0}{2}, \frac{0-4}{2}\right) = (5, -2)$

Let m be the slope of the required line.

$m \times \frac{2}{5} = -1 \Rightarrow m = \frac{-5}{2}$

$\therefore$ Equation of perpendicular bisector of the line segment joining points

$(y + 2) = \left(-\frac{5}{2}\right)(x - 5)$

$\Rightarrow \quad 2y + 4 = -5x + 25$

$\Rightarrow \quad 5x + 2y = 21$

15. *(e)* Let equation of the line parallel to $x + \frac{1}{2}y = \frac{3}{2}$ is

$x + \frac{1}{2}y = \lambda$ ... (i)

Since, Eq. (i) is passing through $(1, 3)$,

$\therefore \quad 1 + \frac{1}{2}(3) = \lambda$

$\Rightarrow \quad \lambda = \frac{5}{2}$

$\therefore$ Required equation of line

$x + \frac{1}{2}y = \frac{5}{2}$

$\Rightarrow \quad 2x + y = 5$

16. *(a)* Given, straight line is $ax + 2ay = 30$

$\Rightarrow \quad \frac{x}{30/a} + \frac{y}{30/2a} = 1$

Since, $\frac{30}{a} = 10 \Rightarrow a = 3$

$\therefore$ y-intercept $= 30/2a = \frac{30}{6} = 5$

17. *(a)* Given, $\cos\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin\alpha = \sqrt{1 - \cos^2\alpha}$

$\sin\alpha = \sqrt{1 - 3/4} = \frac{1}{2}$

$m = \tan\theta = \frac{1}{\sqrt{3}}$

Required equation of line is $y + 2 = \frac{1}{\sqrt{3}}(x - 0)$

$\Rightarrow \quad \sqrt{3}y - x + 2\sqrt{3} = 0$

18. *(b)* Given, equation of circle

$3x^2 + 3y^2 + 6x - 4y - 1 = 0$

$\Rightarrow \quad x^2 + y^2 + 2x - \frac{4y}{3} - \frac{1}{3} = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$$2g = 2 \Rightarrow g = 1$$
$$2f = -\frac{4}{3} \Rightarrow f = -\frac{2}{3} \text{ and } c = -\frac{1}{3}$$

$\therefore$ Radius $= \sqrt{g^2 + f^2 - c}$

$$= \sqrt{(1)^2 + \left(-\frac{2}{3}\right)^2 + \frac{1}{3}}$$
$$= \sqrt{1 + \frac{4}{9} + \frac{1}{3}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

19. *(c)* Required equation of circle as $(-1, 4)$ and $(5, 4)$ are the end points of diameter is

$$(x + 1)(x - 5) + (y - 4)(y - 4) = 0$$
$$\Rightarrow \quad x^2 - 5x + x - 5 + (y - 4)^2 = 0$$
$$\Rightarrow \quad (x - 2)^2 + (y - 4)^2 = 9$$

20. *(a)* The intersecting point of lines $x - y = 5$ and $2x + y = 4$ is $(3, -2)$.

$\therefore$ Centre of circle is $(3, -2)$.

Required equation of circle is

$$(x - 3)^2 + (y + 2)^2 = (5)^2$$
$$\Rightarrow \quad x^2 + 9 - 6x + y^2 + 4 + 4y = 25$$
$$\Rightarrow \quad x^2 + y^2 - 6x + 4y = 12$$

21. *(e)* Let equation of parabola be

$$(y - k)^2 = 4a(x - h)$$

Given, vertex $(h, k) = (-6, 2)$

$$\therefore \quad (y - 2)^2 = 4a(x + 6)$$

This equation is passing through $(-3, 5)$,

$$\therefore \quad (5 - 2)^2 = 4a(-3 + 6)$$
$$\Rightarrow \quad 9 = 4a(3)$$
$$\Rightarrow \quad a = 3/4$$

$\therefore$ Required equation of parabola

$$(y - 2)^2 = 4(3/4)(x + 6)$$
$$\Rightarrow \quad (y - 2)^2 = 3x + 18$$

22. *(a)* Given, centre of the ellipse is $(-2, 1)$.

Let equation of ellipse with centre $(-2, 1)$ is

$$\frac{(x + 2)^2}{a^2} + \frac{(y - 1)^2}{b^2} = 1 \quad \ldots \text{(i)}$$

Since, $(1, 1)$ lies on Eq. (i),

$$\therefore \quad \frac{(3)^2}{a^2} + \frac{(0)^2}{b^2} = 1$$
$$\Rightarrow \quad a^2 = 9$$

And also, $(-2, -1)$ lies on Eq. (i),

$$\therefore \quad \frac{(0)^2}{a^2} + \frac{(-2)^2}{b^2} = 1$$
$$\Rightarrow \quad b^2 = 4$$

$\therefore$ Required equation of ellipse is

$$\frac{(x + 2)^2}{9} + \frac{(y - 1)^2}{4} = 1$$

23. *(c)* Let $P(x, y)$ be the point on the parabola.

Then, the distance of $P(x, y)$ from focus $(3, 0)$

= Distance of $P(x, y)$ from the directrix $x + 3 = 0$

$$\Rightarrow \sqrt{(x - 3)^2 + (y - 0)^2} = \frac{|x + 3|}{\sqrt{(1)^2 + (0)^2}}$$
$$\Rightarrow \sqrt{x^2 + 9 - 6x + y^2} = |x + 3|$$
$$\Rightarrow \quad x^2 + 9 - 6x + y^2 = x^2 + 9 + 6x$$
$$\Rightarrow \quad y^2 = 12x$$

24. *(a)* Given, equation of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Here, $a^2 = 36, b^2 = 16$

We know that,

$$b^2 = a^2(1 - e^2)$$
$$\Rightarrow \quad 16 = 36(1 - e^2)$$
$$\Rightarrow \quad 1 - e^2 = \frac{16}{36} = \frac{4}{9}$$
$$\Rightarrow \quad e^2 = 1 - \frac{4}{9} \Rightarrow e^2 = \frac{5}{9}$$
$$\Rightarrow \quad e = \frac{\sqrt{5}}{3}$$

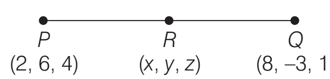
25. *(e)* Distance between foci $(8, 3)$ and $(0, 3)$ is

$$\sqrt{(0 - 8)^2 + (3 - 3)^2} = 8$$
$$\Rightarrow \quad 2ae = 8$$
$$\Rightarrow \quad ae = 4$$
$$\Rightarrow \quad a\left(\frac{4}{3}\right) = 4 \qquad \left(\text{given, } e = \frac{4}{3}\right)$$
$$\Rightarrow \quad a = 3$$

$\therefore$ Length of transverse axis $= 2a = 2 \times 3 = 6$

26. *(d)* Given, $2|PR| = |RQ|$

$$\Rightarrow \quad \frac{|PR|}{|RQ|} = \frac{1}{2}$$



Coordinates of R are

$$\left(\frac{1\times 8+2\times 2}{1+2}, \frac{1\times(-3)+2\times 6}{1+2}, \frac{1\times 1+2\times 4}{1+2}\right)$$

$$=\left(\frac{12}{3}, \frac{9}{3}, \frac{9}{3}\right) = (4, 3, 3)$$

27. *(b)* Given, $|a| = 2$ and $b = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\Rightarrow \quad |b| = \sqrt{(2)^2 + (-1)^2 + (-3)^2} = \sqrt{14}$

$\therefore \quad a \cdot b = |a|\,|b| \cos\theta$

$= 2 \times \sqrt{14} \cdot \cos\frac{\pi}{4}$

$= 2\sqrt{14} \times \frac{1}{\sqrt{2}} = 2\sqrt{7}$

28. *(c)* Given, $a = 5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{k}$ and let $b = \hat{\mathbf{i}}$

$\therefore \quad a \cdot b = |a|\,|b| \cos\alpha$

$\Rightarrow \quad 5 = \sqrt{25 + 9 + 16} \cdot 1 \cdot \cos\alpha$

$\Rightarrow \quad 5 = \sqrt{50} \cdot \cos\alpha$

$\Rightarrow \quad 5 = 5\sqrt{2} \cos\alpha$

$\Rightarrow \quad \cos\alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

29. *(c)* Given, $|a - b| = \sqrt{7}$

$\Rightarrow \quad |a - b|^2 = 7$

$\Rightarrow \quad (a - b) \cdot (a - b) = 7$

$\Rightarrow \quad |a|^2 - 2a \cdot b + |b|^2 = 7$

$\Rightarrow \quad 9 - 2a \cdot b + 16 = 7$

$\Rightarrow \quad 25 - 2a \cdot b = 7$

$\Rightarrow \quad 2a \cdot b = 18$

$\Rightarrow \quad a \cdot b = 9$

30. *(d)* Given, $a \cdot b = -20$

$\Rightarrow \quad (\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} - 2\hat{k}) \cdot (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{k}) = -20$

$\Rightarrow \quad 2 - 3\lambda - 10 = -20$

$\Rightarrow \quad -3\lambda = -20 + 8 = -12$

$\Rightarrow \quad \lambda = 4$

31. *(d)* Given, $a = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \alpha\hat{k}$

$b = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{k}$

$$a \times b = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{k} \\ 1 & -3 & \alpha \\ 1 & -2 & 4 \end{vmatrix}$$

$\Rightarrow \quad a \times b = \hat{\mathbf{i}}(-12 + 2\alpha) - \hat{\mathbf{j}}(4 - \alpha) + \hat{\mathbf{k}}(-2 + 3)$

$\Rightarrow \quad a \times b = \hat{\mathbf{i}}(-12 + 2\alpha) - \hat{\mathbf{j}}(4 - \alpha) + \hat{k}$

$= -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \beta\hat{k}$

$\Rightarrow \quad \beta = 1$

32. *(a)* Since, vectors $\alpha\hat{\mathbf{i}} + (\alpha - 1)\hat{j} + 3\hat{k}$ and $(\alpha + 2)\hat{\mathbf{i}} + \alpha\hat{j} - 2\hat{k}$ are perpendicular.

So, $[\alpha\hat{\mathbf{i}} + (\alpha - 1)\hat{j} + 3\hat{k}]$.

$[(\alpha + 2)\hat{\mathbf{i}} + \alpha\hat{j} - 2\hat{k}] = 0$

$\Rightarrow \alpha(\alpha + 2) + (\alpha - 1)\alpha + 3(-2) = 0$

$\Rightarrow \quad \alpha^2 + 2\alpha + \alpha^2 - \alpha - 6 = 0$

$\Rightarrow \quad 2\alpha^2 + \alpha - 6 = 0$

$\Rightarrow \quad 2\alpha^2 + 4\alpha - 3\alpha - 6 = 0$

$\Rightarrow \quad 2\alpha(\alpha + 2) - 3(\alpha + 2) = 0$

$\Rightarrow \quad (2\alpha - 3)(\alpha + 2) = 0$

$\Rightarrow \quad \alpha = -2, \frac{3}{2}$

33. *(e)* $|u \times v| = |u|\,|v| \sin\frac{\pi}{6}$

$\Rightarrow \quad |u \times v| = 5 \times 4 \times \frac{1}{2} = 10$

34. *(e)* Given, equation of line

$r = \hat{i} + 3\hat{j} + 4\hat{k} + \lambda(2\hat{\mathbf{i}} - \hat{j})$

In cartesian form,

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{0} \quad \ldots \text{(i)}$$

Since, $P(x, 1, 4)$ lies on line (i),

$\therefore \quad \frac{x-1}{2} = \frac{1-3}{-1} = 2$

$\Rightarrow \quad x - 1 = 4$

$\Rightarrow \quad x = 5$

35. *(e)* Equation of plane passing through the point (2, 1, 3) and perpendicular to the vector $4i + 5j + 6k$ is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\Rightarrow \quad 4(x - 2) + 5(y - 1) + 6(z - 3) = 0$

$\Rightarrow \quad 4x + 5y + 6z - 8 - 5 - 18 = 0$

$\Rightarrow \quad 4x + 5y + 6z = 31$

36. *(b)* Given, line

$r = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + t(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{k})$

and plane $2x - 3y - z = 1$

$\Rightarrow (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{k})(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{k}) = 1$

$\therefore$ Angle between line and plane

$$\sin\theta = \frac{(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{k}) \cdot (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})}{|2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{k}| \cdot |3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{k}|}$$

$$= \frac{6-6+1}{\sqrt{4+9+1}\sqrt{9+4+1}} = \frac{1}{14}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{14}\right)$$

37. *(d)* Given, line $\mathbf{r} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + t(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ ... (i)

and plane $2x + 4y + az + 8$ ... (ii)

$$\Rightarrow \quad (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}})\,(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + a\hat{\mathbf{k}}) = 8$$

Since, given line, line (i) is parallel to plane (ii),

$$\therefore \quad \mathbf{n}.\mathbf{b} = 0$$

$$\Rightarrow (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + a\hat{\mathbf{k}}).(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 0$$

$$\Rightarrow \quad 6 + 4 - 2a = 0$$

$$\Rightarrow \quad 10 - 2a = 0$$

$$\Rightarrow \quad a = 5$$

38. *(b)* Angle between given lines

$$\cos\theta = \left|\frac{\mathbf{b}_1 . \mathbf{b}_2}{|\mathbf{b}_1||\mathbf{b}_2|}\right| = \frac{(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}).(3\hat{\mathbf{i}} + \hat{\mathbf{k}})}{\sqrt{4+1+1}\sqrt{9+0+1}}$$

$$= \frac{6-1}{\sqrt{6}\sqrt{10}} = \frac{5}{\sqrt{6}\sqrt{10}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{\sqrt{15}}{6}\right)$$

39. *(a)* Direction ratios of line which is parallel to normal are 3, 2, 1.

$\therefore$ Cartesian equation of the line passing through (7, 5, 3) and having direction ratios 3, 2, 1 is

$$\frac{x-7}{3} = \frac{y-5}{2} = \frac{z-3}{1}$$

40. *(b)* Given, equation of planes

$$2x - y - 3z = 7 \quad \text{... (i)}$$

and $$x + 2y + 2z = 0 \quad \text{... (ii)}$$

$\therefore a_1 = 2, b_1 = -1, c_1 = -3$

and $a_2 = 1, b_2 = 2, c_2 = 2$

Let θ be the angle between given planes,

then, $$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$= \left|\frac{2\times 1 + (-1)(2) + (-3)(2)}{\sqrt{4+1+9}\sqrt{1+4+4}}\right|$$

$$= \left|\frac{-6}{\sqrt{14}\sqrt{9}}\right| = \frac{6}{\sqrt{14}\times 3} = \frac{6\sqrt{14}}{14\times 3} = \frac{\sqrt{14}}{7}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{\sqrt{14}}{7}\right) = \pi - \cos^{-1}\left(-\frac{\sqrt{14}}{7}\right)$$

41. *(a)* Given, $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\mathbf{b} = -2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$\therefore \quad \mathbf{b} - \mathbf{a} = (-2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$= -4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

$\therefore$ Required equation of line

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$$

$$= (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + \lambda(-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

42. *(e)* Total balls = 5 + 3 + 2 + 7 = 17

Number of non-white balls = 5 + 3 + 2 = 10

$$\therefore \text{Required probability} = \frac{^{10}C_4}{^{17}C_4}$$

$$= \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} \times \frac{4\times 3\times 2\times 1}{17\times 16\times 15\times 14} = \frac{3}{34}$$

43. *(a)* Total outcomes = 25×25

and total number of favourable outcomes = 25

$$\therefore \text{Required probability} = \frac{25}{25\times 25} = \frac{1}{25}$$

44. *(c)* Given, $P(A) = 0.2$

$P(B) = 0.55$

and $P(A \cap B) = 0.1$

$$\therefore \quad P(B \cap A') = P(B) - P(A \cap B)$$

$$= 0.55 - 0.1 = 0.45$$

45. *(b)* Given, A = Event that sum of the numbers appearing is 4

$= \{(1, 3), (2, 2), (3, 1)\}$

and B = Event that 3 appears at least once

$= \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (1, 3), (2, 3), (4, 3), (5, 3), (6, 3)\}$

$$A \cap B = \{(1, 3), (3, 1)\}$$

$$\Rightarrow \quad n(A \cap B) = 2$$

$$\therefore \quad P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{2/36}{11/36} = \frac{2}{11}$$

46. *(b)* We know that,

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 7 = \frac{315}{\sum_{i=1}^{8} f_i}$$

$$\Rightarrow \quad \sum_{i=1}^{8} f_i = \frac{315}{7} = 45$$

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47. *(e)* Given, $\sum_{i=1}^{50} x_i = 650$

and $\sum_{i=1}^{50} x_i^2 = 10000$

$$\therefore \quad \text{Variance} = \frac{1}{n}\Sigma x_i^2 - \left(\frac{\Sigma x_i}{n}\right)^2$$
$$= \frac{1}{50} \times 10000 - \left(\frac{650}{50}\right)^2$$
$$= 200 - (13)^2$$
$$= 200 - 169 = 31$$

48. *(c)* We know that,
$$V(X) = E(X^2) - [E(X)]^2$$
$$\Rightarrow \quad E(X^2) = V(X) + [E(X)]^2$$
$$\Rightarrow \quad E(X^2) = 3 + (6)^2$$
$$= 3 + 36 = 39$$

49. *(b)* Given, $f(x) = \frac{4x+3}{x+2}$
$$\Rightarrow \quad y(x+2) = 4x+3$$
$$\Rightarrow \quad yx + 2y = 4x + 3$$
$$\Rightarrow \quad x(y-4) = 3 - 2y$$
$$\Rightarrow \quad x = \frac{3-2y}{y-4}$$
$$\Rightarrow \quad f^{-1}(y) = \frac{3-2y}{y-4}$$
$$\Rightarrow \quad f^{-1}(x) = \frac{3-2x}{x-4}$$
$$\Rightarrow \quad f^{-1}(-2) = \frac{3+4}{-2-4} = -\frac{7}{6}$$

50. *(c)* Given, $f(x)$ is continuous on R.
$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$
$\therefore$ At $x = 1$
$$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h)$$
$$= \lim_{h \to 0} 2(1-h) = 2$$
and $f(1) = 5a - 1$
$$\therefore \quad 5a - 1 = 2$$
$$\Rightarrow \quad 5a = 3$$
$$\Rightarrow \quad a = 3/5$$

51. *(a)* $\lim_{t \to 0} \frac{\sin 2t}{8t^2 + 4t} = \lim_{t \to 0} \frac{\sin 2t}{2t} \times \frac{2t}{8t^2 + 4t}$
$$= \lim_{t \to 0} \frac{\sin 2t}{2t} \times \lim_{t \to 0} \frac{1}{4t+2}$$
$$= 1 \times \frac{1}{2} = \frac{1}{2}$$

52. *(d)* $\lim_{x \to 0} \frac{x}{\sqrt{9-x} - 3}$
$$= \lim_{x \to 0} \frac{x}{\sqrt{9-x} - 3} \times \frac{\sqrt{9-x}+3}{\sqrt{9-x}+3}$$
$$= \lim_{x \to 0} \left(\frac{x}{9-x-9}\right)(\sqrt{9-x}+3)$$
$$= \lim_{x \to 0} (-1)(\sqrt{9-x}+3)$$
$$= (-1)(\sqrt{9-0}+3) = -6$$

53. *(d)* $= \lim_{x \to -2^-} f(x) = \lim_{h \to 0} f(-2-h)$
$$= \lim_{h \to 0} [3(-2-h) + 2]$$
$$= \lim_{h \to 0} (-6 - 3h + 2)$$
$$= -4$$
$$\lim_{h \to -2^+} f(x) = \lim_{h \to 0} f(-2+h)$$
$$= \lim_{h \to 0} [(-2+h)^2 - 3(-2+h) - 1]$$
$$= \lim_{h \to 0} (4 + h^2 - 4h + 6 - 3h - 1)$$
$$= \lim_{h \to 0} (9 + h^2 - 7h) = 9$$

54. *(d)* $\lim_{x \to -3} \frac{x^2 + 16x + 39}{2x^2 + 7x + 3}$ $\left[\therefore \left(\frac{0}{0}\right) \text{form}\right]$
$$= \lim_{x \to -3} \left(\frac{2x+16}{4x+7}\right)$$
$$= \frac{2(-3)+16}{4(-3)+7} = \frac{10}{-5} = -2$$

55. *(c)* Given, $f(x) = 6\sqrt[3]{x^5} = 6x^{5/3}$
$$f'(x) = 6(5/3)x^{2/3}$$
$$= 10x^{2/3} = ax^p$$
$$\therefore p = 2/3$$

56. *(c)* $y = (\tan x)^{\sin x}$

Taking log on both sides,
$$\log y = \log(\tan x)^{\sin x}$$
$$\log y = \sin x \log(\tan x)$$
Differentiating both side w.r.t. x,
$$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \log(\tan x) + \frac{\sin x}{\tan x} \times \sec^2 x$$

$$\frac{dy}{dx} = y[\cos x \cdot \log(\tan x) + \sec x]$$
$$\frac{dy}{dx} = (\tan x)^{\sin x}(\cos x \log(\tan x) + \sec x)$$

Comparing the value of $\frac{dy}{dx}$ with

$(\tan x)^{\sin x} \cdot [\cos x \log(\tan x) + g(x)]$

Hence, $g(x) = \sec x$

57. *(e)* $f(x) = (x^3 + \sin \pi x)^5$

Differentiate w.r.t. x,

$$f'(x) = 5(x^3 + \sin \pi x)^4 (3x^2 + \pi \cos \pi x)$$
$$f'(1) = 5(1 + 0)^4 (3 - \pi) = 5(3 - \pi)$$

58. *(b)* Given, $h(x) = 4x^3 - 5x + 7$ is the derivative of $f(x)$.

$$\lim_{h \to 0} \frac{f(1 + t) - f(1)}{t} = f'(1)$$
$$= h(1)$$
$$= 4(1)^3 - 5(1) + 7$$
$$= 4 - 5 + 7 = 6$$

59. *(b)* $f(x) = \begin{cases} e^x & , \text{if } x \le 1 \\ mx + 6 & , \text{if } x > 1 \end{cases}$

$$f'(x) = \begin{cases} e^x & , \text{if } x \le 1 \\ m & , \text{if } x > 1 \end{cases}$$

As, the function is differentiable at $x = 1$

So, LHD = RHD at $x = 1$

$$e^x = m \text{ at } x = 1$$

Hence, $m = e$

60. *(d)* $\lim_{t \to 0} \frac{\tan^2\left(\frac{\pi}{3} + t\right) - 3}{t}$

Applying L' Hospital Rule,

$$\lim_{t \to 0} \frac{2\tan\left(\frac{\pi}{3} + t\right) \times \sec^2\left(\frac{\pi}{3} + t\right)}{1}$$

Substitute 0 for t,

$$= 2\tan\frac{\pi}{3} \times \sec^2\frac{\pi}{3}$$
$$= 2\sqrt{3} \times 4 = 8\sqrt{3}$$

61. *(d)* Let the equation of the tangent line to the graph of the function be $y = mx + c$.

$$y\text{-intercept} = -10$$
$$\Rightarrow \quad y = mx - 10$$

As, x-intercept is $\frac{5}{3}$, so the tangent of the line passes through $\left(\frac{5}{3}, 0\right)$.

$$0 = \frac{5}{3}m - 10$$
$$\frac{5}{3}m = 10$$
$$m = \frac{30}{5}$$
$$m = 6$$

So, the equation of the tangent line is $y = 6x - 10$ or $f(x) = 6x - 10$

Differentiate w.r.t. x,

$$f'(x) = 6$$
$$f'(3) = 6$$

62. *(d)* Given, equation of curve $4x^2 + 2xy + y^2 = 12$

To find the slope of tangent line of the curve, determine $\frac{dy}{dx}$ at the point $(1, 2)$.

Differentiate the equation of curve w.r.t. x,

$$8x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-(4x + y)}{x + y}$$
$$\left.\frac{dy}{dx}\right|_{(1,\,2)} = -\frac{(4 + 2)}{1 + 2} = -2$$

63. *(c)* Given, $f(x) = \sqrt{x} + 5$ for $1 \le x \le 9$

$$\Rightarrow \quad f'(x) = \frac{1}{2\sqrt{x}}$$

Since, $f(x)$ satisfied Mean Value theorem,

$\therefore f'(c) = \frac{f(b) - f(a)}{b - a}$, where $c \in (1, 9)$

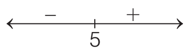
$$\Rightarrow \quad \frac{1}{2\sqrt{c}} = \frac{3 - 1}{9 - 1}$$
$$\Rightarrow \quad \frac{1}{2\sqrt{c}} = \frac{2}{8} = \frac{1}{4}$$
$$\Rightarrow \quad \sqrt{c} = 2 \Rightarrow c = 4$$

64. *(a)* $f'(x) = \frac{x - 5}{\sqrt{x^2 + 4}}$

Critical points can be determined by substituting 0 for $f'(x)$.

$$f'(x) = 0$$
$$\frac{x - 5}{\sqrt{x^2 + 4}} = 0$$

$$x = 5$$



$$f'(x) < 0 \ \forall \ x < 5$$
$$f'(x) > 0 \ \forall \ x > 5$$

The function is increasing at $(5, \infty)$.

65. *(e)* $f(x) = x^2 \log x, \ x > 0$

$$f'(x) = x^2 \times \frac{1}{x} + \log x \times 2x$$
$$= x(1 + 2\log x)$$
$$= x(1 + \log x^2)$$
$$f'(x) = 0$$

As $x \neq 0, 1 + \log x^2 = 0$

$$\log x^2 = -1$$
$$x^2 = e^{-1}$$
$$x = \frac{1}{\sqrt{e}}$$
$$f''(x) = 1 + \frac{2}{x}$$
$$f''\left(\frac{1}{\sqrt{e}}\right) = 1 + 2\sqrt{e} > 0$$

Hence, $f(x)$ is minimum at $x = \frac{1}{\sqrt{e}}$

$$f\left(\frac{1}{\sqrt{e}}\right) = \frac{1}{e}\log e^{-1/2} = \frac{-1}{2e}$$

66. *(a)* Let the edge of cube be x.

$$\frac{dx}{dt} = 2\text{inches/sec}$$

Volume of cube $= x^3$

Differentiate w.r.t. t,

$$\frac{dv}{dt} = 3x^2 \frac{dx}{dt}$$
$$\left.\frac{dv}{dt}\right|_{x=5} = 3 \times 5^2 \times 2$$
$$= 150 \text{ inches}^3\text{/sec}$$

67. *(c)* $\int x^5 e^{1-x^6} dx$

Let $1 - x^6 = t$

$$-6x^5 dx = dt$$
$$x^5 dx = -\frac{1}{6} dt$$
$$\int x^5 e^{1-x^6} dx = -\frac{1}{6}\int e^t \ dt$$
$$= -\frac{1}{6}e^t + C$$
$$= -\frac{1}{6}e^{1-x^6} + C$$

68. *(a)* $\int (5 - 4x) \ e^{-x} dx$

Using the integration by parts method,

$$= (5-4x)\int e^{-x} dx - \int \frac{d}{dx}[(5-4x)\int e^{-x} dx] dx$$
$$= -(5-4x)e^{-x} - \int (-4)e^{-x}(-1) \ dx$$
$$= (4x - 5)e^{-x} - 4(-e^{-x}) + C$$
$$= (4x - 5)e^{-x} + 4e^{-x} + C$$
$$= e^{-x}[4x - 5 + 4] + C$$
$$= e^{-x}(4x - 1) + C$$

69. *(b)* $\int \frac{\cos(\tan x)}{\cos^2 x} dx = \int \cos(\tan x).\sec^2 x \, dx$

Let $\tan x = t$.

$$\sec^2 x \ dx = dt$$

$\therefore \quad \int \cos t \, dt = \sin t + C$

$$= \sin(\tan x) + C$$

70. *(e)* Let $I = \int \frac{1}{e^{2x} - 1} dx$

$$= \int \frac{1}{(e^x)^2 - 1} dx$$

Let $\quad e^x = t$

$$e^x dx = dt$$
$$dx = \frac{1}{t} dt$$

$\therefore \quad I = \int \frac{1}{(t^2 - 1)} \times \frac{1}{t} dt$

$$I = \int \frac{1}{t(t-1)(t+1)} dt \quad \ldots \text{(i)}$$

Applying partial fraction,

$$\frac{1}{t(t-1)(t+1)} = \frac{A}{t} + \frac{B}{t-1} + \frac{B}{t+1}$$
$$\frac{1}{t(t-1)(t+1)} = \frac{A(t-1)(t+1) + Bt(t+1) + C(t-1)t}{t(t-1)(t+1)}$$

Comparing the coefficient of t^2, t and constant,

$$1 = t^2(A + B + C) + t(B - C) + (-A)$$
$$-A = 1 \Rightarrow A = -1$$
$$B - C = 0 \Rightarrow B = C$$
$$A + B + C = 0$$
$$-1 + 2B = 0$$

$$B = \frac{1}{2} \text{ and } C = \frac{1}{2}$$

$$\int \frac{1}{t(t-1)(t+1)}\,dt = \int -\frac{1}{t}dt + \frac{1}{2}\int \frac{1}{t-1}\,dt + \frac{1}{2}\int \frac{1}{t+1}\,dt$$

$$= -\log|t| + \frac{1}{2}\log|t-1| + \frac{1}{2}\log|t+1| + C$$

$$= -\log|t| + \frac{1}{2}\log|t^2-1| + C$$

$$= -\log e^x + \frac{1}{2}\log|e^{2x}-1| + C$$

$$= -x + \frac{1}{2}\log|e^{2x}-1| + C$$

Hence, $I = \frac{1}{2}\log|e^{2x}-1| - x + C$

71. *(b)* Let $I = \int \sin 2x \cos x\,dx$

$$I = \int 2\sin x \cos^2 x\,dx$$

Let $\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

$$I = -2\int t^2 dt$$

$$= -\frac{2t^3}{3} + C$$

$$= \frac{-2}{3}\cos^3 x + C$$

72. *(e)* Let $I = \int \frac{1}{(1+\cot^2 x)\sin^2 x}\,dx$

$$= \int \frac{\operatorname{cosec}^2 x\,dx}{(1+\cot^2 x)}$$

$$= \int \frac{\operatorname{cosec}^2 x}{\operatorname{cosec}^2 x}\,dx$$

$$= \int 1\,dx$$

$$= x + C$$

Hence, $I = x + C$

73. *(b)* Let $I = \frac{4x^9}{x^{10}-10}\,dx$

Let $x^{10} - 10 = t$

$$10x^9\,dx = dt$$

$$x^9 dx = \frac{1}{10}dt$$

$$I = \frac{4}{10}\int \frac{1}{t}\,dt$$

$$= \frac{4}{10}\log|t| + C$$

$$= \frac{2}{5}\log|x^{10}-10| + C$$

Hence, $I = \frac{2}{5}\log|x^{10}-10| + C$

74. *(a)* Let $I = \int_0^{\sqrt{3}} \frac{6}{9+x^2}\,dx$

Using the formula, $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$$I = 6\left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^{\sqrt{3}}$$

$$= 2\left[\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) - \tan^{-1}(0)\right]$$

$$= 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - 0$$

$$= 2\times\frac{\pi}{6} = \frac{\pi}{3}$$

75. *(e)* Let $I = \int_{-5}^{5}(4-|x|)\,dx$

$$I = \int_{-5}^{5} 4dx - \int_{-5}^{5}|x|\,dx$$

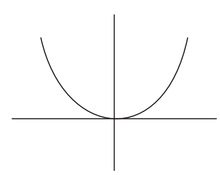
$$= [4x]_{-5}^{5} - \left[\int_{-5}^{0}(-x)\,dx + \int_0^5 x\,dx\right]$$

$$= (4\times 5) - [4\times(-5)] - \left[\left[-\frac{x^2}{2}\right]_{-5}^{0} + \left[\frac{x^2}{2}\right]_0^5\right]$$

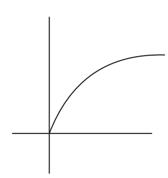
$$= (20+20) - \left[\frac{25}{2} + \frac{25}{2}\right]$$

$$= 40 - 25 = 15$$

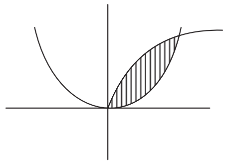
76. *(b)* The graph of the curves $y = x^2$ is represented as



and the graph of the curve $y = \sqrt{x}$ is represented as



The area bounded by the two curves can be represented as



The intersecting point of the curve can be determined by solving both equations.

$$y = y^4$$
$$y^4 - y = 0$$
$$y(y^3 - 1) = 0$$

Either $y = 0$ or $y = 1$

If, $y = 0$, then $x = 0$ and if, $y = 1$, then $x = 1$

Area of the bounded region

$$= \int_0^1 (\sqrt{x} - x^2)dx$$
$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$$
$$= \frac{2}{3} - \frac{1}{3}$$
$$= \frac{1}{3}\text{square units}$$

77. *(d)* Let $I = \int_0^2 \frac{x^2}{(x^3 + 1)^2}\,dx$

Let $x^3 + 1 = t$

$3x^2dx = dt$

$\Rightarrow \quad x^2dx = \frac{1}{3}dt$

If $x \to 0$, then $t \to 1$

If $x \to 2$, then $t \to 9$

$$I = \frac{1}{3}\int_1^9 \frac{1}{t^2}dt$$
$$I = \frac{1}{3}\left[-\frac{1}{t}\right]_1^9$$
$$= \frac{1}{3}\left[-\frac{1}{9} - (-1)\right]$$
$$= \frac{1}{3}\left[1 - \frac{1}{9}\right]$$
$$= \frac{1}{3} \times \frac{8}{9} = \frac{8}{27}$$

78. *(b)* Let $I = \int_{\pi/8}^{3\pi/8} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx$ …(i)

Using the property, $\int_a^b f(x)^{dx} = \int_a^b f(a + b - x)\,dx$

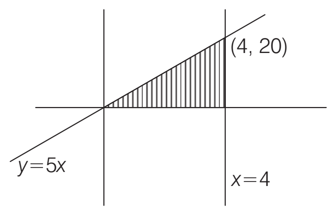
$$I = \int_{\pi/8}^{3\pi/8} \frac{\sin^4\left(\frac{\pi}{8} + \frac{3\pi}{8} - x\right)}{\sin^4\left(\frac{\pi}{8} + \frac{3\pi}{8} - x\right) + \cos^4\left(\frac{\pi}{8} + \frac{3\pi}{8} - x\right)}\,dx$$

$$I = \int_{\pi/8}^{3\pi/8} \frac{\cos^4 x}{\cos^4 x + \sin^4 x}\,dx \quad \text{… (ii)}$$

Adding Eqs. (i) and (ii),

$$2I = \int_{\pi/8}^{3\pi/8} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x}\,dx$$
$$2I = \int_{\pi/8}^{3\pi/8} 1\,dx$$
$$2I = [x]_{\pi/8}^{3\pi/8}$$
$$2I = \frac{3\pi}{8} - \frac{\pi}{8}$$
$$2I = \frac{2\pi}{8}$$
$$\Rightarrow \quad I = \frac{\pi}{8}$$

79. *(a)* The area bounded by $y = 5x$, x-axis and $x = 4$ can be represented as



Area of the region $= \int_0^4 y\,dx$

$$= \int_0^4 5x\,dx = 5\left[\frac{x^2}{2}\right]_0^4 = 5 \times 8 = 40$$

80. *(a)* The differential equation

$$y - xy' = x^2 + y^2$$
$$xy' = y - x^2 - y^2$$
$$y' = \frac{y}{x} - x - \frac{y^2}{x}$$

Let $y = vx$ or $\frac{y}{x} = v$

Differentiate w.r.t. x,

$$y' = x \cdot \frac{dv}{dx} + v$$

$$v + x\frac{dv}{dx} = v - x - \frac{v^2x^2}{x}$$

$$x\frac{dv}{dx} = -x - v^2x$$

$$\frac{dv}{dx} = -(1+v^2)$$

$$\int \frac{1}{1+v^2}\,dv = \int -dx$$

$$\tan^{-1} v = -x + c$$

$$v = \tan(c - x)$$

$$\frac{y}{x} = \tan(c - x)$$

Hence, $y = x\tan(c - x)$

81. *(b)* Given, differential equation

$$xy' + 2y - 7x^3 = 0$$

$$\frac{dy}{dx} + 2\frac{y}{x} = 7x^2$$

Here, $P = \frac{2}{x}$

$$\text{IF} = e^{\int P\,dx} = e^{\int \frac{2}{x}dx} = e^{2\log|x|} = x^2$$

82. *(e)* $4xy + 12x + (2x^2+3)y' = 0$

$$4x(y+3) + (2x^2+3)y' = 0$$

$$\frac{d}{dx}(y+3)(2x^2+3) = 0$$

$$(y+3)(2x^2+3) = C$$

83. *(e)* The constraints of LPP are $x + 2y \le 10$ and $6x + 3y \le 18$.

Here, the points (0, 6), (4, 3), (5, 7) and (1, 7) does not lie on the constraints $x + 2y \le 10$.

Hence, (1, 3) lies on the feasible region.

$$x + 2y \le 10 \Rightarrow 1 + 2\times 3 = 7 \le 10$$

$$6x + 3y \le 18 \Rightarrow 6\times 1 + 3\times 3 = 12 \le 18$$

84. *(c)* $f : [-4, 2] \to R$

$$f(x) = \sqrt{16 - x^2}$$

Let $y = \sqrt{16-x^2} \Rightarrow y \ge 0$

Squaring both sides,

$\Rightarrow \quad y^2 = 16 - x^2$

$\Rightarrow \quad x^2 = 16 - y^2$

$$x = \pm\sqrt{16 - y^2}$$

$$16 - y^2 \ge 0 \Rightarrow y \in [0, 4]$$

The range of the function $f(x) = \sqrt{16-x^2}$ is [0, 4].

85. *(d)* Let $f(x) = x^2$ and $g(x) = \sqrt{9+x}$

$$fog(x) = f[g(x)] = f[\sqrt{9+x}] = 9 + x$$

$$gof(x) = g[f(x)] = g[x^2] = \sqrt{9+x^2}$$

$$(fog - gof)(x) = [9 + x - \sqrt{9+x^2}]$$

$$(fog - gof)(4) = 9 + 4 - \sqrt{9+16} = 13 - 5 = 8$$

86. *(b)* Given, $n(A) = 24$, $n(A\cap B) = 8$

$$n(U) = 63$$

$$n(A' \cup B') = n(A\cap B)' \quad \text{[De-Morgan's Law]}$$

$$= n(U) - n(A\cap B)$$

$$= 63 - 8 = 55$$

87. *(a)* $f(x) = [x]\ \forall\ x \in R$

$$f(-4.6) = [-4.6] = -5$$

$$f(2.7) = [2.7] = 2$$

88. *(c)* It is given that, $m * n = \frac{m+n}{3}$

$$\frac{7}{2} * \frac{5}{2} = \frac{\frac{7}{2}+\frac{5}{2}}{3} = \frac{\frac{12}{2}}{3} = 2$$

89. *(d)* Given, $f(x) = 7 - 3x$

Let $x_1 = x_2\ \forall\ x_1, x_2 \in R$

$$-3x_1 = -3x_2$$

$$7 - 3x_1 = 7 - 3x_2$$

$$f(x_1) = f(x_2)$$

Thus, the function is one-one.

Let $y = 7 - 3x$

$$x = \frac{7-y}{3}$$

$$f(x) = 7 - 3\left(\frac{7-y}{3}\right) = y$$

Thus, the function is onto.

Hence, the function $f(x) = 7 - 3x$ is one-one and onto $\forall\ x \in R$.

90. *(a)* Given, $R = \{(0, 0), (1, 1), (0, 1), (2, 2), (1, 2)\}$ on set $\{0, 1, 2\}$.

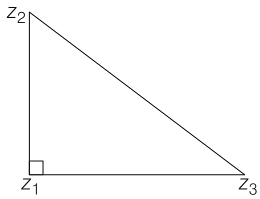
The reaction R is reflexive as $(0, 0), (1, 1), (2, 2) \in R$

The relation R is not symmetric as $(0, 1) \in R$ but $(1, 0) \notin R$.

The relation R is not transitive as (0, 1) and (1, 2) $\in R$ but $(0, 2) \notin R$.

Hence, the relation R is only reflexive.

91. *(c)* Since, the segment joining z_1 and z_2 is perpendicular to segment joining z_1 and z_3.



$\therefore \quad (|z_2 - z_3|)^2 = (|z_1 - z_2|)^2 + (|z_1 - z_3|)^2$

$= (5)^2 + (12)^2$

$= 25 + 144 = 169$

$\therefore \quad |z_2 - z_3| = \sqrt{169} = 13$

92. *(d)* $\frac{z}{i} = 11 - 13i$

$z = 11i - 13i^2$

$z = 13 + 11i$

$\bar{z} = 13 - 11i$

$z + \bar{z} = 13 + 11i + 13 - 11i = 26$

93. *(e)* $\alpha = 2 - 3i$

$\alpha^2 = (2 - 3i)^2 = 4 - 9 - 12i = -5 - 12i$

α is the root of the equation

$\alpha^2 - 4\alpha + k = 0$

$-5 - 12i - 4(2 - 3i) + k = 0$

$-5 - 12i - 8 + 12i + k = 0$

$k = 13$

Now, the equation can be written as

$z^2 - 4z + 13 = 0$

$\alpha + \beta = 4$ and $\alpha\beta = 13$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (4)^2 - 2 \times 13$

$= 16 - 26 = -10$

94. *(d)* $z = 2 - i\sqrt{3}$

$|z| = \sqrt{2^2 + (-\sqrt{3})^2} = \sqrt{4 + 3} = \sqrt{7}$

$|z^4| = |z|^4 = (\sqrt{7})^4 = 49$

95. *(c)* Given, $z = \frac{2 + i}{3 - i}$

$z = \frac{(2 + i)(3 + i)}{(3 - i)(3 + i)}$

$z = \frac{(6 - 1) + i(3 + 2)}{9 + 1}$

$z = \frac{5 + 5i}{10}$

$z = \frac{1}{2} + \frac{1}{2}i$

Hence, imaginary part of $\frac{2 + i}{3 - i} = \frac{1}{2}$

96. *(b)* Let $z = x + iy$,

$z + iz = (x - y) + i(x + y)$,

$iz = -y + ix$

Area of triangle $= \frac{1}{2}\begin{vmatrix} x & y & 1 \\ x - y & x + y & 1 \\ -y & x & 1 \end{vmatrix} = 128$

$x(x + y - x) - y(x - y + y) + 1$

$(x^2 - xy + xy + y^2) = 256$

$xy - xy + x^2 + y^2 = 256$

$x^2 + y^2 = 256$

$\Rightarrow \quad |z|^2 = 256$

$\therefore \quad |z| = 16$

97. *(b)* $z = \frac{(p + 2i)}{(p - i)} \times \frac{(p + i)}{(p + i)}$

$= \frac{(p^2 - 2)}{p^2 + 1} + \frac{i(2p + p)}{p^2 + 1}$

$\text{Re}(z) = \frac{p^2 - 2}{p^2 + 1} = \frac{1}{2}$

$2p^2 - 4 = p^2 + 1$

$p^2 = 5$

$\Rightarrow \quad p = \sqrt{5}$

98. *(e)* $\sqrt{(-25)} + 3\sqrt{(-4)} + 2\sqrt{(-9)}$

$= 5i + 3 \times 2i + 2 \times 3i$

$= 5i + 6i + 6i = 17i$

99. *(c)* $\sum_{K=5}^{36} \frac{1}{(K^2 - K)}$

Using the partial fraction method,

$\frac{1}{K(K - 1)} = \frac{A}{K} + \frac{B}{K - 1}$

$1 = AK - A + BK$

$A + B = 0$ and $-A = 1$

$\Rightarrow \quad A = -1$

$-1 + B = 0$

$B = 1$

$$\frac{1}{K}(K-1) = -\frac{1}{K} + \frac{1}{K-1}$$

$$\sum_{K=5}^{36} \frac{1}{K^2 - K} = \sum_{K=5}^{36}\left(-\frac{1}{K} + \frac{1}{K-1}\right)$$

$$= \left(-\frac{1}{5} + \frac{1}{4}\right) + \left(-\frac{1}{6} + \frac{1}{5}\right) + \left(-\frac{1}{7} + \frac{1}{6}\right) + \ldots\ldots + \left(-\frac{1}{36} + \frac{1}{35}\right)$$

$$= \frac{1}{4} - \frac{1}{36} = \frac{2}{9}$$

100. *(b)* Given, $a_1 = 3, a_n = 39$

$$S_n = 210$$

$$\Rightarrow \frac{n}{2}[a + a_n] = 210$$

$$\frac{n}{2}[3 + 39] = 210$$

$$n = \frac{210 \times 2}{42}$$

$$n = 10$$

101. *(a)* Given, AP $\Rightarrow$ 5, 8, 11, …

$$n = 305$$

$$a + (n-1)d = 305$$

$$5 + (n-1)(8-5) = 305$$

$$5 + 3n - 3 = 305$$

$$3n = 303$$

$$n = 101$$

102. *(c)* Given, $a_1 = 1, a_3 + a_5 = 90$

$$\Rightarrow ar^2 + ar^4 = 90$$

$$r^2 + r^4 = 90 \quad (\because a = 1)$$

$$r^4 + r^2 - 90 = 0$$

$$r^4 + 10r^2 - 9r^2 - 90 = 0$$

$$r^2 = -10 \text{ or } r^2 = 9$$

$$r^2 = 9 \quad (\text{as, } r^2 \neq -10)$$

$$r = \pm 3$$

Hence, the positive value of r is 3.

103. *(d)* Let the number of terms in AP be n.

$$a_n - a = 632, d = 4$$

$$a + (n-1)d - a = 632$$

$$(n-1)4 = 632$$

$$n - 1 = 158$$

$$n = 159$$

104. *(e)* Given, $a_{10} = 15$ and $a_{12} = 21$

$$a_{10} = 15 \Rightarrow a + 9d = 15 \quad \ldots \text{(i)}$$

$$a_{12} = 21 \Rightarrow a + 11d = 21 \quad \ldots \text{(ii)}$$

Subtracting Eq. (i) from Eq. (ii),

$$2d = 6$$

$$d = 3$$

105. *(d)* Given, $a = 3$ and $r = 2$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_8 = \frac{3(2^8 - 1)}{2 - 1}$$

$$= 3 \times (256 - 1)$$

$$= 3 \times 255$$

$$= 765$$

106. *(d)* Let $E_1 \rightarrow$ event that people got vaccinated

$E_2 \rightarrow$ event that people does not vaccinate

$A \rightarrow$ event that people get Covid-19 infection

We have, $P(E_1) = 45\%$

$$P(E_2) = 55\%$$

$$P(A/E_2) = 0.4$$

$\therefore$ Required probability,

$$P(E_2 \cap A) = P(E_2).P(A/E_2)$$

$$= \frac{55}{100} \times 0.4$$

$$= 0.22$$

107. *(c)* The required number of ways of forming a committee is

$${}^8C_5 \times {}^5C_3$$

$$\Rightarrow {}^8C_3 \times {}^5C_2$$

$$\frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \frac{5 \times 4}{1 \times 2}$$

$$= 56 \times 10 = 560$$

108. *(d)* The number of subsets which contain at most 4 elements are

$${}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$$

$$= 1 + 9 + \frac{9 \times 8}{1 \times 2} + \frac{9 \times 8 \times 7}{1 \times 2 \times 3} + \frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4}$$

$$= 10 + 36 + 84 + 126$$

$$= 256$$

109. *(d)* Given, ${}^{p+q}P_2 = 42$

$$\Rightarrow \frac{(p+q)!}{(p+q-2)!} = 42$$

$$\Rightarrow (p+q)(p+q-1) = 42 \quad \ldots \text{(i)}$$

$$(p+q)^2 - (p+q) - 42 = 0$$

$$\therefore \quad p+q = 7, -6$$

Thus, $p+q = 7, p+q \neq = -6$

and ${}^{p-q}P_2 = 20$

$$\Rightarrow \frac{(p-q)!}{(p-q-2)!} = 20$$

$$\Rightarrow (p-q)(p-q-1) = 20 \quad \ldots \text{(ii)}$$

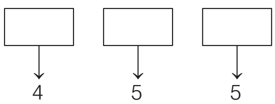
$$(p-q)^2 - (p-q) - 20 = 0$$

$$p-q = 5, -4$$

$$p-q = 5, p-q \neq -4$$

Solving the two equations $p+q = 7$ and $p-q = 5$, we get $p = 6, q = 7$

110. *(b)* Formation of 3-digit numbers from the digits 0, 2, 3, 5, 7.



Hence, the number of 3-digit numbers

$$= 4 \times 5 \times 5 = 100$$

111. *(c)* $T_{r+1} = {}^9C_r (3x^3)^{9-r} (-x^2)^r$

$$= {}^9C_r (3)^{9-r} (x)^{27-3r} (-1)^r (x)^{2r}$$

$$= {}^9C_r (3)^{9-r} (-1)^r x^{27-r}$$

Compare the power of x,

$$27 - r = 22$$

$$r = 5$$

112. *(e)* $T_{r+1} = {}^{20}C_r \, x^{20-r} \left(\frac{2}{x^3}\right)^r$

$$= {}^{20}C_r \, x^{20-r} \frac{(2)^r}{x^{3r}}$$

$$= {}^{20}C_r (2)^r x^{20-4r}$$

Comparing the power of x with zero,

$$20 - 4r = 0$$

$$r = 5$$

The term independent of x is $\binom{20}{5} 2^5$.

113. *(e)* $A + B = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -1 & 3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 & 6 \\ 2 & 1 & 4 \end{bmatrix}$$

114. *(e)* Let $\Delta = \begin{vmatrix} 4 & 4^2 & 4^3 \\ 3 & 3^2 & 3^3 \\ 2 & 2^2 & 2^3 \end{vmatrix}$

$$= 4 \times 3 \times 2 \begin{vmatrix} 1 & 4 & 16 \\ 1 & 3 & 9 \\ 1 & 2 & 4 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 = R_3 - R_1$

$$= 24 \begin{vmatrix} 1 & 4 & 16 \\ 0 & -1 & -7 \\ 0 & -2 & -12 \end{vmatrix}$$

$$= 24(12 - 14)$$

$$= -48$$

115. *(a)* $\begin{vmatrix} 1 & 2 & 1 \\ 0 & x & -3 \\ 2 & -1 & x \end{vmatrix} = 0$

Expand along C_1,

$$1(x^2 - 3) + 2(-6 - x) = 0$$

$$x^2 - 3 - 12 - 2x = 0$$

$$x^2 - 2x - 15 = 0$$

$$x^2 - 5x + 3x - 15 = 0$$

$$(x - 5)(x + 3) = 0$$

$$x = 5, -3$$

116. *(e)* $AB = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$

$$B = A^{-1} \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 12 - 10 & 9 - 8 \\ -4 + 5 & -3 + 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

117. *(a)* Let $A = \begin{bmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & \lambda & 0 \end{bmatrix}$

If A is non-singular, then $|A| \neq 0$.

$$-2(-\lambda) - 1(4) \neq 0$$
$$2\lambda - 4 \neq 0$$
$$\Rightarrow \quad 2\lambda \neq 4$$
$$\Rightarrow \quad \lambda \neq \frac{4}{2}$$
$$\therefore \quad \lambda \neq 2$$

118. *(e)* Given, $\begin{vmatrix} x-1 & 1 & 1 \\ 2 & x-1 & 2 \\ 1 & x+2 & x-1 \end{vmatrix}$

$$= ax^3 + bx^2 + cx + d$$

$$(x-1)[x^2 + 1 - 2x - 2x - 4] - 2[2x - 2 - 2] + 1[2x + 4 - x + 1]$$
$$= ax^3 + bx^2 + cx + d$$
$$(x-1)[x^2 - 4x - 3] - 2[2x - 4] + 1[x + 5]$$
$$= ax^3 + bx^2 + cx + d$$
$$x^3 - 4x^2 - 3x - x^2 + 4x + 3 - 4x + 8 + x + 5$$
$$= ax^3 + bx^2 + cx + d$$
$$x^3 - 5x^2 - 2x + 16 = ax^3 + bx^2 + cx + d$$

Comparing the constant part to determine the value of d.

Hence, $d = 16$

119. *(b)* Given, $-13 \leq x \leq 5$

$$-13 + 4 \leq x + 4 \leq 5 + 4$$
$$-9 \leq x + 4 \leq 9$$

Thus, $|x + 4| \leq 9$

Compare this expression with $|x - a| \leq b$ to determine the value of a and b.

Hence, $a = -4$, $b = 9$

120. *(a)* Given, $5(4x + 6) < 25x + 10$

$$5(4x + 6) < 5(5x + 2)$$
$$4x + 6 < 5x + 2$$
$$6 - 2 < 5x - 4x$$
$$4 < x$$

Hence, $x \in (4, \infty)$.